SOLID STATE ELECTRONIC DEVICES EE339

INDEX

INTRODUCTION (see also PeriodicTableAndAtoms.pdf)

In a **crystal**, atoms are arranged in a periodic array called a **lattice structure**. Some types of lattice structure are 1) **simple cubic** having one atom at each corner of a cube for a total internal content of 1 atom/cube, 2) **body-centered cubic** having an additional atom in the center of the cube, 2 atoms/cube, 3) **face-centered cubic** having an atom at each corner and another in the center of each outer face, 4 atoms/cube, 4) **diamond** having a face-centered structure with 4 additional internal atoms, 8 atoms/cube. The length of each side of the cube is the **lattice constant**, denoted by a, and expressed in units of angstroms $(1\text{Å} = 10^{-8}$ cm). We are sometimes interested in the **nearest neighbor** distance, the center-to-center distance between the two closes atoms in a cube, expressed in terms of a. In the diamond lattice, this is ¼ the length of the diagonal connecting the most distant opposing corners of the cube, $a\sqrt{3}/4$.

Crystals are **tailored** in order to control 1) electron transport, 2) absorption and radiation of light.

Elemental semiconductors have a **diamond lattice structure**. **Binary compounds** composed of combinations from

3-layer close-packed plane

groups II-V and II-VI have a zinc blende structure similar to diamond structure.
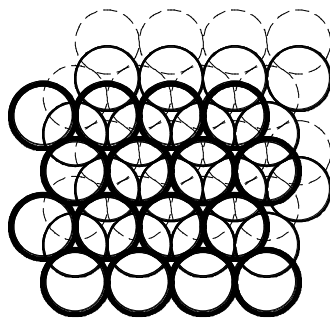
Silicon is not a metal because there is a gap between the bonding states and the non-bonding states. Two states of the same energy are said to be degenerate. Semiconductors have a small **energy gap**. Insulators have a large energy gap. In metals there is overlap between bonding states.

Intrinsic semiconductors such as pure Si and Ge, have an identical number of electrons in the conduction band and holes in the valence band. At 0K, all electrons return to the valence band.

Doping is the controlled introduction of impurities into a pure substance. This generally results in an **extrinsic semiconductor**, i.e. a material having either a surplus of electrons (n-type) or a surplus of holes (p-type).

Single crystal means that the atoms are arranged in a single array. **Polycrystalline** means that atoms are in multiple arrays, having **grain boundaries** between them. **Amorphous** means the atoms are arranged with no definite form.

Bravais (bra-VAY) is a lattice with one atom per cell.

A **cubic close-packed plane** has layers of atoms arranged in 3 offset arrays as shown in the illustration at right.

Epitaxy is the technique of growing an oriented, single-crystal layer on a substrate layer having an identical or similar lattice structure. Epitaxy methods include 1) CVD Chemical Vapor Deposition (also used for polycrystaline and amorphous deposition), 2) LPE Liquid-Phase Epitaxy - growth from a melt, 3) MBE Molecular Beam Epitaxy - elements are evaporated in a vacuum.

Quantum mechanics is a theory of matter that is based on the concept of the possession of wave properties by elementary particles, that affords a mathematical interpretation of the structure and interactions of matter on the basis of these properties, and that incorporates within it quantum theory and the uncertainty principle — called also *wave mechanics*.

LIGHT

Luminescence – the property of light emission

Photoluminescence – radiation resulting from the recombination of carriers excited by photon absorption. Some materials react rapidly to the excitation being turned off; this process is called **fluorescence**. In other materials, emission continues long after excitation is removed; these processes are called **phosphorescence**, the materials are **phosphors**.

Cathodoluminescence – radiation resulting from the recombination of excited carriers created by high-energy electron bombardment.

Electroluminescence – radiation resulting from the recombination of carriers excited by the introduction of current into the sample. The **injection electroluminescence** effect occurs when an electric current causes the injection of minority carriers into regions of the semiconductor crystal where they can recombine with majority carriers, resulting in recombination radiation—used in LEDs.

P-N JUNCTION FABRICATION

Grown – A junction is created by abruptly changing the doping during the growth of the crystal.

Alloyed – A pellet on one material is melted into an oppositely-doped material and a *regrown* crystal region forms at the junction.

Diffused – The crystal is heated to about 1000ºC causing atoms to move out of their lattice positions. Doping atoms in a gaseous state outside the material move into the vacated positions by diffusion. This methods offers precision and is implemented in integrated circuit fabrication.

Ion Implantation – A beam of impurity ions is accelerated to a kinetic energy in the range of several KeV to several MeV and fired into the crystal. The depth at which the ions come to rest can be controlled by the kinetic energy level. Used in integrated circuits. Damage to the lattice caused by ion collisions can be repaired by *annealing*, or heating of the crystal.

BE CAREFUL WITH UNITS!

In this course, energy is usually expressed in eV rather than J, and distance is often cm rather than m. Be sure to be consistent with units in a formula, paying attention to the units of constants as well. Some formulas just do not work with units of eV; calculate energy in J and convert to eV.

PHOTON

The quantity of energy gained or lost when an electron moves from one energy level to the next, a quantized unit of light energy whose magnitude is dependent upon frequency. The amount of energy is equal to Plank's constant multiplied by the radiation frequency.

$$1\,\text{photon} = h\nu = E_2 - E_1 \quad [\text{J}]$$

Ev ELECTRON VOLT

The energy gained when an electron moves to a point with a 1 volt greater potential $[1.60 \times 10^{-19}\,\text{J}]$. To convert from joules to electron volts, divide J by q.

WAVELENGTH

$\boxed{c = \lambda\nu}$ $1\text{Å} = 10^{-10}\text{m}$	c = speed of light 2.998×10^8 m/s λ = wavelength [m] ν = (nu) radiation frequency [Hz] Å = (angstrom) unit of distance equal to 10^{-10} m m = (meters)

WAVELENGTH SPECTRUM

BAND	METERS	ANGSTROMS
Longwave radio	1 - 100 km	10^{13} - 10^{15}
Standard Broadcast	100 - 1000 m	10^{12} - 10^{13}
Shortwave radio	10 - 100 m	10^{11} - 10^{12}
TV, FM	0.1 - 10 m	10^9 - 10^{11}
Microwave	1 - 100 mm	10^7 - 10^9
Infrared light	0.8 - 1000 μm	8000 - 10^7
Visible light	360 - 690 nm	3600 - 6900
violet	360 nm	3600
blue	430 nm	4300
green	490 nm	4900
yellow	560 nm	5600
orange	600 nm	6000
red	690 nm	6900
Ultraviolet light	10 - 390 nm	100 - 3900
X-rays	5 - 10,000 pm	0.05 - 100
Gamma rays	100 - 5000 fm	0.001 - 0.05
Cosmic rays	< 100 fm	< 0.001

PHOTOELECTRIC EFFECT

In order to escape from a metal being bombarded by light, an electron requires an amount of energy $q\Phi$ and emits an energy $h\nu$. The **work function** Φ, is the minimum amount of energy needed to cause an electron to leave the metal. It is a constant for each metal. Tungsten: 4.5 eV.

$$\boxed{E_m = h\nu - q\Phi}$$ (must know)

E_m = maximum energy of emitted electrons [J]

h = Planck's constant, 6.63×10^{-34} J-s

ν = (nu) frequency [Hz]

q = electron charge 1.6022×10^{-19} [c]

$q\Phi$ = (phee) the work function of the metal [eV]

CENTRIPETAL FORCE

The electrostatic force between an electron and its nucleus equals the centripetal force.

$$\boxed{-\frac{q^2}{Kr^2} = -\frac{m\mathbf{v}^2}{r}}$$

$$\mathbf{v} = \frac{q^2}{Kn\hbar}$$

q = electron charge 1.6022×10^{-19} c

$K = 4\pi\varepsilon_0$ [$4\pi \times 8.85 \times 10^{-12}$ F/m]

r = orbital radius [m]

m = electron mass [9.11×10^{-31} kg], replace with effective mass $m*$ in crystaline structures

$\mathbf{v}$ = electron velocity [m/s]

n = orbit of the electron [integer]

$\hbar$ = Planck's constant divided by 2π [J-s]

ELECTRON ENERGY

$$K.E. = \frac{1}{2}mv^2 = \frac{mq^4}{2K^2n^2\hbar^2}$$

$$P.E. = -\frac{q^2}{Kr_n}mv^2 = -\frac{mq^4}{K^2n^2\hbar^2}$$

$$E_n = K.E. + P.E. = -\frac{mq^4}{2K^2n^2\hbar^2}$$

$K.E.$ = kinetic energy [J]

$P.E.$ = potential energy [J]

E_n = total energy of an electron in the nth orbit [J]

m = electron mass [9.11×10^{-31} kg] for the hydrogen atom; for other problems use m_n*, the effective mass typical of the semiconductor used.

v = velocity [m/s]

$K = 4\pi\varepsilon_0$ for the hydrogen atom or $4\pi\varepsilon_0\varepsilon_r$ for other atoms where ε_r is the relative dielectric constant of the semiconductor material [F/m]

h = Planck's constant, 6.63×10^{-34} J-s

q = charge on the electron 1.6022×10^{-19} [c]

n = orbit of the electron [integer]

$\hbar$ = Planck's constant divided by 2π [J-s]

LYMAN, BALMER, & PASCHEN SERIES

Describe groups of lines that appear in an emissions spectrum.

Lyman: $\nu = cR\left(\dfrac{1}{1^2} - \dfrac{1}{\mathbf{n}^2}\right)$, $\quad \mathbf{n}$ = 2, 3, 4, …

Balmer: $\nu = cR\left(\dfrac{1}{2^2} - \dfrac{1}{\mathbf{n}^2}\right)$, $\quad \mathbf{n}$ = 3, 4, 5, …

Paschen: $\nu = cR\left(\dfrac{1}{3^2} - \dfrac{1}{\mathbf{n}^2}\right)$, $\quad \mathbf{n}$ = 4, 5, 6, …

ν = (nu) frequency [Hz]

c = speed of light 2.998×10^{10} cm/s

R = Rydberg constant, 109,678 cm^{-1}

f- DEPENDENT WAVE FUNCTION

$$\boxed{\Phi_m(\phi) = \frac{1}{\sqrt{2\pi}}e^{jm\phi}}$$

Φ = (Fee) work function potential [V]

ϕ = (fee) potential [V]

m = quantum number [integer]

BOHR MODEL

Niels Bohr constructed a model for the hydrogen atom based on the mathematics of planetary systems. He made the following postulates:

1. Electrons exist in certain stable, circular orbits about the nucleus.
2. The electron may shift to an orbit of higher or lower energy, gaining or losing an amount of energy equal to the difference in energy levels by absorbing or emitting 1 photon of energy $h\nu$.
3. The angular momentum $\mathbf{r_q}$ of the electron in an orbit is always an integral multiple of Planck's constant divided by 2π. $\rho_0 = Nh/2\pi = N\hbar$.

BOHR ELECTRON ORBIT RADIUS

The electron of the hydrogen atom may occupy one of several orbits **n** of radius r.

$$rm^* q^2 = 4\pi\varepsilon_0\varepsilon_r \mathbf{n}^2\hbar^2$$

r = orbit radius [m]
m^* = effective mass of the semiconductor material [kg]
q = electron charge 1.6022×10^{-19} [C]
ε_0 = permittivity of free space [F/m]
ε_r = relative permittivity of the material [constant]
n = orbit, **n** = 1 is ground state [integer]
$\hbar$ = Planck's constant divided by 2π [J-s]

$\mathbf{e_r}$ RELATIVE PERMITTIVITY

The ratio of the permittivity of the material to the **permittivity of free space** ($\varepsilon_0 = 8.85 \times 10^{-12}$ F/m). The permittivity of the material is then $\varepsilon_0\varepsilon_r$. The relative permittivities of some common materials are:

Si 11.8, Ge 16, GaAs 13.2

SCHRÖDINGER WAVE EQUATION

In three dimensions: $-\dfrac{\hbar^2}{2m}\nabla^2\Psi + V\Psi = -\dfrac{\hbar}{j}\dfrac{\partial\Psi}{\partial t}$

where $\nabla^2\Psi = \dfrac{\partial^2\Psi}{\partial x^2} + \dfrac{\partial^2\Psi}{\partial y^2} + \dfrac{\partial^2\Psi}{\partial z^2}$

$\hbar$ = Planck's constant divided by 2π [J-s]
m = quantum number [integer]
∇ = total energy of an electron in the nth orbit [J]
$\Psi(x)$ = the wave function, having space and time dependencies
$j = \sqrt{-1}$
t = time [s]
x, y, z = position coordinates [m]

refer to **WaveEquation.pdf** for more information

EFFECTIVE MASS OF AN ELECTRON

The **effective mass** of an electron is simply an adjustment to the standard electron mass that accounts for the influence of the **lattice structure**. This enables the same formulas to work for different elements.

$$m^* = \frac{\hbar^2}{d^2 E / d\mathbf{k}^2}$$

m^* = effective mass of an electron (n), hole (p). [kg]
$\hbar$ = Planck's constant divided by 2π [J-s]
d = derivative
E = electron energy scale [J]
$\mathbf{k}$ = wave vector [m^{-1}]

The effective mass for several semiconductors is given below in terms of the free electron rest mass m_0. These are average values appropriate for density of states calculations only.

	Ge	Si	GaAs
m_n^*	$0.55m_0$	$1.1m_0$	$0.067m_0$
m_p^*	$0.37m_0$	$0.56m_0$	$0.48m_0$

E DONOR BINDING ENERGY

The amount of energy required to excite a donor electron to the conduction band.

$$E = \frac{m^* q^4}{8\left(\varepsilon_0\varepsilon_r\right)^2 h^2}$$

E = donor binding energy [J]
m^* = effective mass of the semiconductor material [kg]
q = electron charge 1.60×10^{-19} [C]
ε_0 = permittivity of free space [F/m]
ε_r = relative permittivity of the material [constant]
h = Planck's constant, 6.63×10^{-34} J-s

n_i INTRINSIC CONCENTRATION OF HOLES AND ELECTRONS

An **intrinsic** semiconductor is a perfect semiconductor crystal (no impurities). The number of electrons in the conduction band **equals the number of holes** in the valence band. At 0K, all electrons return to the valence band and there are no excess carriers. The opposite of intrinsic is **extrinsic** and is achieved by doping.

$$\boxed{p_0 n_0 = n_i^{\,2}} \qquad n_i = \sqrt{N_c N_v}\; e^{-E_g/2kT}$$

p_0 = concentration of holes in the valence band at equilibrium [cm^{-3}] This value is often taken to be equal to N_a, the acceptor doping concentration, since at room temperature it may be assumed that each acceptor will host an electron from the valence band.

n_0 = concentration of electrons in the conduction band at equilibrium [cm^{-3}] This value is often taken to be equal to N_d, the donor doping concentration, since at room temperature it may be assumed that each donor has contributed an electron to the conduction band.

n_i = intrinsic concentration of electrons in the conduction band (constant for a particular material at temperature) [cm^{-3}]

N_c = effective density of states in the conduction band [cm^{-3}]

N_v = effective density of states in the valence band [cm^{-3}]

E_g = band gap energy [eV]

k = Boltzmann's constant 8.62×10^{-5} eV/K

T = absolute temperature [K]

kT = constant (0.02586 when $T = 300$ and k is in eV/K)

Some intrinsic concentrations @ 300K:
- Si: $n_i = 1.5 \times 10^{10}$ cm^{-3}
- Ge: $n_i = 2.5 \times 10^{13}$ cm^{-3}
- GaAs: $n_i = 2 \times 10^6$ cm^{-3}

E_g ENERGY BAND GAP

The difference in potential between the conduction band and the valence band in electron volts eV. The energy band gaps of selected semiconductor materials are listed below:

Si	1.11 eV	GaSb	0.7 eV	ZnSe	2.7 eV
Ge	0.67 eV	InP	1.35 eV	CdS	2.42 eV
AlAs	2.16 eV	InAs	0.36 eV	CdSe	1.73 eV
GaP	2.26 eV	InSb	0.18 eV	PbS	0.37 eV
GaAs	1.43 eV	ZnS	3.6 eV	PbSe	0.27 eV

$f(E)$ FERMI-DIRAC DISTRIBUTION FUNCTION

For **intrinsic materials** (such as pure Si crystal with all valence electrons covalently bonded) the **Fermi level E_F** is near the center of the **energy band gap**. For n-type material the Fermi level is off-center toward the conduction band and for p-type material the Fermi level is off-center toward the valence band.

$$\boxed{f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}}$$

$f(E)$ = Fermi-Dirac distribution function, the distribution of electrons over a range of allowed energy levels at thermal equilibrium, having a value from 0 to 1.

E = an available energy state [eV]

E_F = Fermi level—the energy level at the center of distribution of the free electron population [eV]

k = Boltzmann's constant 8.62×10^{-5} eV/K

T = absolute temperature [K]

kT = constant (0.02586 when $T = 300$ and k is in eV/K)

E_c = the energy level at the conduction band [eV]

E_v = the energy level at the valence band [eV]

| Intrinsic semiconductor | n-type semiconductor | p-type semiconductor |
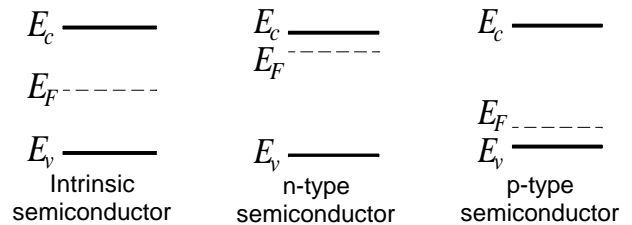
THE BOLTZMANN DISTRIBUTION

For two discrete energy levels E_1 and E_2 with $E_2 > E_1$:

$$\boxed{\frac{n_2}{n_1} = e^{-(E_2 - E_1)/kT}}$$

n_1 = number of atoms in state E_1

n_2 = number of atoms in state E_2

kT = constant (0.02586 when $T = 300$ and k is in eV/K)

N_c, N_v EFFECTIVE DENSITY OF STATES

The *effective density of states* is the concentration of locations that may host free electrons in the conduction band (c) or free holes in the valence band (v) in units of cm^{-3}. This should not be confused with the density of states which are actually occupied, called the *carrier concentration* n_0 or p_0.

$$N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

$$N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

m^* = effective mass of an electron [kg]

h = Planck's constant, 6.63×10^{-34} J-s

k = Boltzmann's constant 1.38×10^{-23} J/K

T = 300 K at room temperature

n_0, p_0 EQUILIBRIUM CARRIER CONCENTRATION

The concentration of free electrons (n_0) or holes (p_0) at the edge of the conduction band (n), or valence band (p) in units of [cm^{-3}] at equilibrium:

$$n_0 = N_c e^{(E_F - E_c)/kT}$$

$$p_0 = N_v e^{(E_v - E_F)/kT}$$

$$n_0 = n_i e^{(E_F - E_i)/kT}$$

$$p_0 = n_i e^{(E_i - E_F)/kT}$$

$$p_0 n_0 = n_i^2$$

n_0 = concentration of electrons in the conduction band at equilibrium [cm^{-3}]

n_i = intrinsic concentration of electrons in the conduction band (constant for a particular material at temperature) [cm^{-3}]

p_0 = concentration of holes in the valence band at equilibrium [cm^{-3}]

N_c = effective density of states in the conduction band [cm^{-3}]

N_v = effective density of states in the valence band [cm^{-3}]

E_c = the conduction band energy level [eV]

E_i = the intrinsic energy level [eV]

E_F = Fermi level—the energy level which is the center of distribution of the free electron population [eV]

kT = constant (0.02586 when T = 300 and k is in eV/K)

SPACE CHARGE NEUTRALITY

The sum of the holes (p_0) and ionized donor atoms (N_d^+) equals the sum of the negative charges (n_0) and ionized acceptor atoms (N_a^-). It is expected that all donors and acceptors are ionized at room temperature.

$$p_0 + N_d^+ = n_0 + N_a^-$$

Certain techniques are employed when working with this equation, e.g. if the material is n-type, then the value of N_d^+ will be high and p_0 can be taken to be zero. Also, the relation $n_0 p_0 = n_i^2$ can be used to reduce the number of unknowns.

ELECTRON BONDING IN SEMICONDUCTORS

Where there are four **nearest neighbors** in the lattice structure, atoms from **group IV** of the periodic table, such as Si and Ge, having 4 electrons in the outer (**valence**) band, form strong (**covalent**) bonds by sharing these electrons with their neighbors. When the material is doped with (**donor**) atoms having more than 4 electrons in the valence band, the excess electrons which do not become covalently bonded are free to participate in conduction. Conversely, when the material is doped with (acceptor) atoms having fewer than 4 electrons in the valence band, holes and unbonded electrons are available for conduction.

TEMPERATURE DEPENDENCE OF CARRIER CONCENTRATIONS

$$n_i(T) = 2 \left(\frac{2pkT}{h^2} \right)^{3/2} \left(m_n * m_p * \right)^{3/4} e^{-E_g/2kT}$$

n_i = intrinsic concentration of electrons in the conduction band (constant for a particular material at temperature) [cm^{-3}]

k = Boltzmann's constant 8.62×10^{-5} eV/K

T = absolute temperature [K]

h = Planck's constant, 6.63×10^{-34} J-s or 4.14×10^{-15} eV

m_n* = effective mass of an electron [kg]

m_p* = effective mass of a hole [kg]

E_g = band gap energy [J, eV]

THE EINSTEIN RELATION

$$\frac{D}{\mu} = \frac{kT}{q}$$

D = diffusion coefficient [cm^2/s]

μ = electron or hole mobility [$cm^2/V\text{-}s$] p8.

kT/q = constant [V] (0.02586 V when T = 300 and k is in J/K)

q = electron charge 1.60×10^{-19} [C]

$\mathbf{s}$ CONDUCTIVITY

$$\sigma = qn\mu_n$$

σ = (sigma) conductivity [$(\Omega\text{-}cm)^{-1}$]

q = electron charge 1.6022×10^{-19} [C]

n = concentration of electrons [cm^{-3}]

p = concentration of holes [cm^{-3}]

μ_n = electron mobility [$cm^2/V\text{-}s$] p8.

μ_p = hole mobility [$cm^2/V\text{-}s$]

μ_n ELECTRON MOBILITY AND DRIFT

Electron mobility is the ease with which electrons drift in the material, or the average particle **drift velocity** per unit of electric field ($\text{cm/s/V/cm} \rightarrow \text{cm}^2\text{/V-s}$). There is an upper limit to the velocity, called **saturation velocity** v_s, that is caused by scattering, about 10^7 cm/s for silicon. See EINSTEIN RELATION p7.

$$\mu_n = \frac{q\bar{t}}{m_n*} = \frac{\langle \mathbf{v}_x \rangle}{\mathscr{E}_x}$$

q = electron charge 1.6022×10^{-19} [C]

μ_n = electron mobility [$\text{cm}^2\text{/V-s}$]

μ_p = hole mobility [$\text{cm}^2\text{/V-s}$]

$\mathscr{E}_x$ = electric field applied in the x-direction [V/cm]

$\langle \mathbf{v}_x \rangle$ = average particle drift velocity [cm/s]

	μ_n [cm^2/V-s]	μ_p [cm^2/V-s]
Silicon Si	1350	480
Germanium Ge	3900	1900
Gallium Arsenide GaAs	8500	400

J CURRENT DENSITY

$$J_x = q\left(n\mu_n + p\mu_p\right)\mathscr{E}_x = \sigma\mathscr{E}_x$$

If an electric field is present in addition to the carrier gradient, the current densities will each have a drift component and a diffusion component:

$$J_n(x) = \underbrace{q\mu_n n(x)\mathscr{E}(x)}_{\text{drift}} + \underbrace{qD_n \frac{dn(x)}{dx}}_{\text{diffusion}}$$

$$J_p(x) = \overbrace{q\mu_p p(x)\mathscr{E}(x)}^{} - \overbrace{qD_p \frac{dp(x)}{dx}}^{}$$

J_n, J_p = current density due to electrons, holes [A/cm^2]

q = electron charge 1.6022×10^{-19} [C]

n, p = concentration of electrons, holes [cm^{-3}]

μ_n, μ_p = electron mobility, hole mobility [cm^2/V-s] p8.

$\mathscr{E}_x$ = electric field in the x-direction [V/cm]

D_n, D_p = diffusion coefficient for electrons, holes [cm^2/s]

σ = (sigma) conductivity [(Ω-cm)$^{-1}$]

INTENSITY OF LIGHT TRANSMITTED THROUGH A SEMICONDUCTOR

$$I_t = I_0 e^{-\alpha l}$$

I_t = intensity of transmitted light [watts or photons/cm^2-s]

I_0 = light source intensity [watts or photons/cm^2-s]

α = absorption coefficient [cm^{-1}]

l = sample thickness [cm]

DRIFT AND DIFFUSION CURRENTS

The two basic processes of current conduction are diffusion due to a carrier gradient, and drift in an electric field. Charge carriers (electrons or holes) move by **diffusion** from areas of high carrier concentration to areas of low carrier concentration. Since the amount of current is proportional to the *carrier gradient* rather than the number of carriers, minority carriers may contribute significantly to the diffusion current. The **recombination** of electrons and holes as carriers pass through a material can create a carrier gradient and produce diffusion current. **Drift current** is the result of electrons in random motion being influenced by an electric field. Although electrons are moving in many directions, the net movement will be *against* the electric field, holes move *with* the field. The result is current flow *with* the electric field. Drift current flows in response to an applied voltage.

$\mathscr{E}(x)$ ELECTRIC FIELD

The electric field $\mathscr{E}(x)$ is equal to the change in voltage in the *x*-direction. An electric field occurs when there is a gradient in the doping concentration, and is related to the change in the intrinsic energy level of the semiconductor material. The direction of an electric field is *uphill* in a band diagram. Electrons *drift* down the slope, while holes *drift* in the field direction.

An electric field may also be the result of a semiconductor junction, see page 11.

$$\mathscr{E}(x) = -\frac{dV(x)}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$

$$\mathscr{E}(x) = \frac{D_p}{\mu_p}\frac{1}{p(x)}\frac{dp(x)}{dx}$$
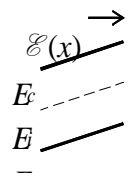
$\mathscr{E}(x)$ = electric field as a function of location [V/cm]

V = voltage [V]

q = electron charge 1.6022×10^{-19} [C]

E_i = the intrinsic energy level [eV]

D_p = diffusion coefficient for holes [cm^2/s]

μ_p = hole mobility [cm^2/V-s] p8.

$P(x)$ = distribution of holes [cm^{-3}]

D_n, D_p DIFFUSION COEFFICIENT

The net motion of electrons (or holes) due to diffusion is in the direction of decreasing electron (or hole) concentration. see EINSTEIN RELATION p7.

$$D_n = \frac{\bar{l}^2}{2\bar{t}}$$

D_n = diffusion coefficient for electrons [cm²/s]

$\bar{l}$ = the mean free path; differential distance [cm]

$\bar{t}$ = the mean free time; differential time [s]

DIFFUSION COEFFICIENT and ELECTRON MOBILITY for intrinsic semiconductors

	D_n (cm²/s)	D_p (cm²/s)	μ_n (cm²/V-s)	μ_p (cm²/V-s)
Ge	100	50	3900	1900
Si	35	12.5	1350	480
GaAs	220	10	8500	400

DIFFUSION EQUATION

For electrons:

$$\frac{\partial \delta n}{\partial t} = D_n \frac{\partial^2 \delta n}{\partial x^2} - \frac{\delta n}{\tau_n}$$

For holes:

$$\frac{\partial \delta p}{\partial t} = D_p \frac{\partial^2 \delta p}{\partial x^2} - \frac{\delta p}{\tau_p}$$

δn, δp = excess electron, hole concentration [cm⁻³]

D_n, D_p = diffusion coefficient for electrons, holes [cm²/s]

x = distance in the (opposite) material p, n [cm]

τ_n, τ_p = recombination lifetime for electrons, holes (applies to minority carriers only) [s]

The **general solution** for the diffusion equation for holes is:

$$\delta p(x) = C_1 e^{x/L_p} + C_2 e^{-x/L_p}$$

Since all holes eventually combine (in a wide *n*-region), $\delta p=0$ at $x=\infty$ so $C_1=0$. At $x=0$, $\delta p=\Delta p$ by definition, so $C_2=\Delta p$. Therefore the **final solution** is: (see EXCESS CARRIERS)

$$\delta p(x) = \Delta p\, e^{-x/L_p}$$

DIFFUSION LENGTH

$$L_n \equiv \sqrt{D_n t_n} \qquad L_p \equiv \sqrt{D_p \tau_p}$$

L_p, L_n = diffusion length; the average distance a hole, or electron, diffuses before recombining; the distance at which the excess hole (or electron) distribution is reduced to $1/e$ of its value at the point of injection (carrier *injection* always implies minority carriers) [cm]

D_n, D_p = diffusion coefficient for electrons, holes [cm²/s]

τ_n = recombination lifetime (applies to minority carriers only) [s]

$\mathbf{f}_n$ ELECTRON FLUX DENSITY

The rate of electron flow per unit area.

$$\phi_n(x_0) = \frac{\bar{l}}{2\bar{t}}(n_1 - n_2) \qquad \phi_n(x) = -D_n \frac{dn(x)}{dx}$$

n = concentration of electrons [cm⁻³]

$\bar{l}$ = the mean free path in the *x* direction [cm]

$\bar{t}$ = the mean free time [s]

D_n = diffusion coefficient for electrons [cm²/s]

$\mathbf{d}p$ EXCESS CARRIERS

Excess carriers are charge carriers (holes or electrons) in excess of quantities found at **thermal equilibrium**. They may be created by optical excitation, electron bombardment, or injected with an applied voltage.

$$\delta p(x_n) = \Delta p\, e^{-x_n/L_p}$$

δp = excess hole concentration [cm⁻³]

Δp = excess hole concentration at the transition [cm⁻³]

x_n = distance along the junction [cm]

L_p = diffusion length; the average distance a hole, or electron, diffuses before recombining; the distance at which the excess hole distribution is reduced to $1/e$ of its value at the point of injection (carrier *injection* always implies minority carriers) [cm]

$\mathbf{d}n(t)$ RECOMBINATION OF EXCESS ELECTRON-HOLE PAIRS (EHPs)

$$\delta n(t) = \Delta n\, e^{-t/\tau_n}$$

$\delta n(t)$ = excess carrier concentration at time *t* [cm⁻³]

Δn = excess carrier concentration at *t*=0 [cm⁻³]

t = time [s]

τ_n = recombination lifetime (applies to minority carriers only) [s]

V_0 CONTACT POTENTIAL

The difference in intrinsic energy levels in the **p** and **n** regions.

$$V_0 = \frac{kT}{q} \ln \frac{p_p}{p_n}$$

$$V_0 = E_{ip} - E_{in}$$

p_p, p_n = equilibrium concentration of holes in the boundary region of the **p** material, of the **n** material [cm^{-3}]. For the **n** material, this might just be the intrinsic concentration n_i, unless there are also acceptors present. In the **p** material this could just be the acceptor concentration N_A, assuming that the intrinsic concentration is small in comparison and there are no donor atoms that would give up electrons to the acceptors.

kT/q = constant (0.02586 when $T = 300$ and k is in J/K) (refer to pn JUNCTION section, page 10.)

p-n STEP JUNCTION

For a step junction (an abrupt transition), the donors and acceptors give up all of their donor electrons and holes to the adjacent region so that $p_p = N_a$ and $n_n = N_d$. The effect of acceptor concentration in the p region is to bring the Fermi energy level close to the valence band, and the effect of donor concentration in the n region is to bring the Fermi level close to the conduction band. When the p and n regions are brought together, the Fermi levels must be equal at equilibrium. See Band Diagram below.

$$p_p = n_i e^{(E_{ip} - E_{Fp})/kT}$$

$$n_n = n_i e^{(E_{Fn} - E_{in})/kT}$$

$$\frac{p_p}{p_n} = \frac{n_n}{n_p} = e^{qV_0/kT}$$

p_p = concentration of holes in the boundary region of the **p** material [cm^{-3}]

n_n = concentration of electrons in the boundary region of the **n** material [cm^{-3}]

n_i = intrinsic concentration of electrons [cm^{-3}]

E_{ip}, E_{in} = intrinsic energy level in the **p** region, **n** region [eV]

E_{Fp}, E_{Fn} = Fermi energy level in the **p** region, **n** region [eV]

kT = constant (0.02586 when $T = 300$ and k is in eV/K)

N_a = concentration of acceptors (atoms that may gain an electron) [cm^{-3}]

N_d = concentration of donors (atoms that may lose an electron) [cm^{-3}]

V_0 = contact potential [eV]

EQUILIBRIUM BAND DIAGRAM

In a p-n junction at equilibrium, the Fermi energy level of the p-material is equal to the Fermi energy level of the n-material. Refer to the relations under **pn STEP JUNCTION** and **V_0 CONTACT POTENTIAL** above to calculate band gap values shown.
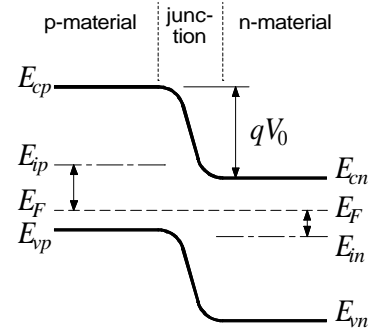
E_{cp} = the p-material conduction band energy level [eV]

E_{ip} = the p-material intrinsic energy level [eV]

E_F = the Fermi energy level [eV]

E_{vp} = the p-material valence band energy level [eV]

E_{cn} = the n-material conduction band energy level [eV]

E_{in} = the n-material intrinsic energy level [eV]

E_{vn} = the n-material valence band energy level [eV]

W BOUNDARY WIDTH

A p-n junction is shown with $N_d > N_a$, hence the **transition region** (also called the *depletion region*) extends further into the p material.
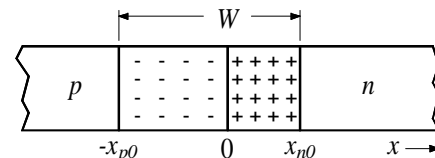
$$W = \left[\frac{2\varepsilon V_0}{q} \left(\frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2} = \left[\frac{2\varepsilon V_0}{q} \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \right]^{1/2}$$

$$= \left[\frac{2\varepsilon kT}{q^2} \left(\ln \frac{N_a N_d}{n_i^2} \right) \left(\frac{1}{N_a} + \frac{1}{N_d} \right) \right]^{1/2}$$

W = boundary width, width of the transition region [cm]

ε = permittivity [F/cm]

N_a = concentration of acceptors (atoms that may gain an electron) [cm^{-3}]

N_d = concentration of donors (atoms that may lose an electron) [cm^{-3}]

V_0 = contact potential [eV]

q = electron charge 1.60×10^{-19} [C]

CHARGE DENSITY WITHIN THE TRANSITION REGION

The number of charges in the x_p region is equal to the number of charges in the x_n region due to **space charge neutrality**. If the magnitude of the charge density is greater in the n material (i.e. there is a higher doping concentration in the n material), then the transition region extends further into the p material in order to balance the charge, as illustrated in the diagram below.

(compare with **W BOUNDARY WIDTH** section previous)

Charge density $\mathrm{C/cm}^3$

ELECTRIC FIELD STRENGTH WITHIN THE TRANSITION REGION

(refer to **W BOUNDARY WIDTH** section previous)
The total area bounded by the function and the *x*-axis is equal to $-V_0$.

$$\mathscr{E}_0 = -\frac{q}{\varepsilon} N_d x_{n0} = -\frac{q}{\varepsilon} N_a x_{p0}$$

$\mathscr{E}_0$ = maximum value of the electric field (occurs at the junction) [V/cm]

$\mathscr{E}(x)$ = electric field [V/cm]

q = electron charge 1.6022×10^{-19} [C]

N_d, N_a = concentration of donors, acceptors [cm^{-3}]

x_{no}, x_{po} = penetration of the transition area into the n region, p region [cm]

W BOUNDARY WIDTH WITH BIAS

$$W = \left[\frac{2e(V_0 - V)}{q} \left(\frac{N_a + N_d}{N_a N_d} \right) \right]^{1/2}$$
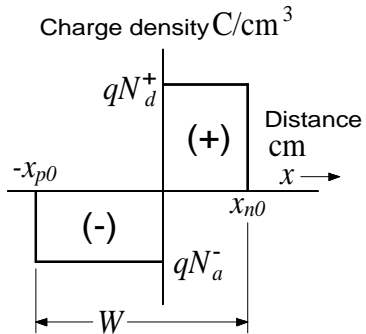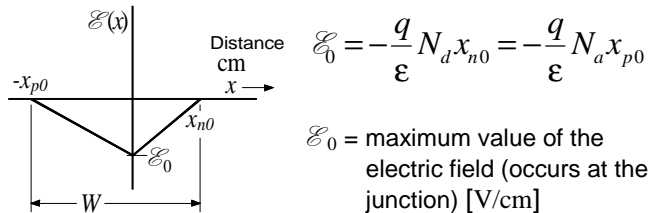
(refer to **W BOUNDARY WIDTH** section previous)

x_{n0}, x_{p0} PENETRATION OF THE TRANSITION AREA INTO THE n & p REGIONS

$$x_{n0} = \frac{N_a}{N_a + N_d} W = \left\{ \frac{2\varepsilon V_0}{q} \left[\frac{N_a}{N_d (N_a + N_d)} \right] \right\}^{1/2}$$

$$x_{p0} = \frac{N_d}{N_a + N_d} W = \left\{ \frac{2\varepsilon V_0}{q} \left[\frac{N_d}{N_a (N_a + N_d)} \right] \right\}^{1/2}$$

$$W = x_{p0} + x_{n0}$$

x_{no} = penetration of the transition area into n region [cm]

x_{po} = penetration of the transition area into p region [cm]

(refer to **W BOUNDARY WIDTH** section p. 10)

Q TOTAL CHARGE ACROSS THE JUNCTION

There must be an equal number of charges on either side of the junction within the transition area. This is called **space charge neutrality**. Since the charge carriers are quickly swept across the junction by the electric field, charges in the transition area are due primarily to uncompensated donors and acceptors.

(refer to **W BOUNDARY WIDTH** section p. 10)

$$Q_+ = Q_- = qAx_{n0}N_d = qAx_{p0}N_a$$

A = the junction area [cm^2]

q = electron charge 1.6022×10^{-19} [C]

x_{no} = penetration of the transition area into the n region [cm]

$N(x,t)$ DOPING BY DIFFUSION (GAS)

The impurity concentration N_0 is held constant at the surface during the diffusion process.

$$N(x,t) = N_0 \, \mathrm{erfc}\left(\frac{x}{2\sqrt{Dt}} \right)$$

$N(x,t)$ = the impurity distribution [cm^{-3}]

N_0 = volume concentration of impurities maintained outside the surface of the material during diffusion [cm^{-3}]

erfc = something called the *complementary error function*

x = penetration level in the material [cm] (typically converted to μm)

D = the diffusion coefficient for the impurity [cm^2/s]

t = the duration of the diffusion process [s]

$N(x,t)$ DOPING BY DIFFUSION LAYER

A layer of impurities is applied to the surface before diffusion. No impurities escape from the material during the diffusion process.

$$N(x,t) = \frac{N_s}{\sqrt{\pi D t}} e^{-\left(x/2\sqrt{Dt}\right)^2}$$

N_S = area concentration of impurities in the surface layer [atoms/cm^2]

see above for other variable definitions

I DIODE EQUATION

$$I = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)\left(e^{qV/kT}-1\right) = I_0\left(e^{qV/kT}-1\right)$$

and for the non-ideal diode: $\boxed{I = I_0\left(e^{qV/nkT}-1\right)}$

I = diode current [A]

q = electron charge 1.6022 × 10^{-19} [C]

A = the junction area [cm^2]

D_p, D_n = diffusion coefficient for holes, electrons [cm^2/s]

L_p, L_n = diffusion length; the average distance a hole, or electron, diffuses before recombining; the distance at which the excess hole (or electron) distribution is reduced to $1/e$ of its value at the point of injection (carrier *injection* always implies minority carriers) [cm]

p_n = concentration of holes in the boundary region of the **n** material [cm^{-3}]

n_p = concentration of electrons in the boundary region of the **p** material [cm^{-3}]

kT = constant (0.02586 when T = 300 and k is in eV/K)

I_0 = saturation current [A]

n = ideality factor 0 < n < 1

I_0 REVERSE SATURATION CURRENT

$$I_0 = qA\left(\frac{D_p}{L_p}p_n + \frac{D_n}{L_n}n_p\right)$$

C CAPACITANCE OF P-N JUNCTIONS

There are basically two types of capacitance associated with a junction:

1. **junction capacitance** or **transition region capacitance** or **depletion layer capacitance**. This is the capacitance due to the dipole in the transition region (the area containing two oppositely charged regions merging at the junction). This type is dominant under reverse-bias conditions. Capacitance varies with voltage since the width of the transition region varies with voltage.

$$C_j = \frac{\varepsilon A}{W} = \varepsilon A\left[\frac{q}{2\varepsilon(V_0-V)}\frac{N_d N_a}{N_d+N_a}\right]^{1/2}$$

2. **charge storage capacitance** or **diffusion capacitance**. This is the capacitance due to the lagging behind of voltage as current changes. This type is dominant under forward-bias conditions. Charge storage capacitance can be a serious limitation in high frequency circuits involving forward-biased p-n junctions.

Stored charge:
$$Q_p = I\tau_p = qA\,\Delta p_n L_p$$
$$= qAL_p p_n e^{qV/kT}$$

Capacitance due to small changes in the stored charge, i.e. **charge storage capacitance:**
$$C_s = \frac{dQ_p}{dV} = \frac{q^2}{kT}AL_p p_n e^{qV/kT}$$
$$= \frac{q}{kT}I\tau_p$$

C_j JUNCTION CAPACITANCE

The capacitance of an **asymmetrically doped junction** is determined by only one of the doping concentrations. C_j is a *voltage-variable* capacitance.

$$\boxed{C_j = \frac{A}{2}\left[\frac{2q\varepsilon}{V_0-V}N_d\right]^{1/2}}$$

For a p$^+$-n junction,
$N_a \ll N_d$, $\;x_{n0} \approx W$

C_j = junction capacitance [F]

A = the junction area [cm^2]

q = electron charge 1.6022 × 10^{-19} [C]

ε = permittivity [F/cm]

V_0 = contact potential [eV]

$(V_0 - V)$ = barrier height [eV]

N_d = concentration of donors (atoms that may lose an electron) [cm^{-3}]

THE p$^+$-n JUNCTION

The p material is more heavily doped so that the depletion layer width W extends primarily into the n side.

$$C_j \propto V_r^{-\mathbf{n}} \text{ for } V_r >> V_0 \qquad N_a << N_d,$$

$$N_d(x) = Gx^{\mathbf{m}} \quad \mathbf{n} = 1/(\mathbf{m}+2) \qquad x_{n0} \approx W$$

C_j = junction capacitance [F]

$\mathbf{n}$ = doping gradient factor

    $\mathbf{n}$ = 1/3 for a **linearly graded** junction

    $\mathbf{n}$ = 1/2 for an **abrupt** junction

    $\mathbf{n}$ > 1/2 for a **hyperabrupt** junction

    $\mathbf{n}$ = 2 for a **varacter** device

G = a constant

V_r =

V_0 = contact potential [eV]

N_d = concentration of donors (atoms that may lose an electron) [cm^{-3}]

DIRECT RECOMBINATION

The direct recombination of electron and hole pairs occurs when electrons fall from the conduction band to empty holes in the valence band. Light is produced. The net rate of change in the conduction band electron concentration is the thermal generation rate minus the recombination rate:

$$\frac{dn(t)}{dt} = \alpha_r n_i^2 - \alpha_r n(t)p(t)$$

α_r = constant of recombination proportionality [cm^{-3}]

n_i = intrinsic concentration of electrons in the conduction band [cm^{-3}]

n = concentration of electrons [cm^{-3}]

p = concentration of holes [cm^{-3}]

If excess carrier concentrations are small and the material is extrinsic, we can say:

$$\frac{d\delta n(t)}{dt} = -\alpha_r p_0 \delta n(t),$$

$$\text{where} \quad \delta n(t) = \Delta n e^{-\alpha_r p_0 t} = \Delta n e^{-t/\tau_n}$$

$$\text{and} \quad \tau_n = \frac{1}{\alpha_r(n_0 + p_0)} \approx (\alpha_r p_0)^{-1}$$

$\delta n(t)$ = concentration of excess electrons [cm^{-3}]

Δn = concentration of excess electrons at t = 0 [cm^{-3}]

τ_n = recombination lifetime, i.e. the average time an electron spends in the energy band before recombination [s]

The excess majority carriers decay at the same rate as the minority carriers, so $\tau_n = \tau_p$. If $\Delta p = \Delta n$, then $\delta p(t) = \delta n(t)$.

INDIRECT RECOMBINATION, TRAPPING

The majority of recombination events in indirect materials, such as column IV semiconductors, occur via recombination levels within the band gap. These energy levels are called **trapping centers** and may be denoted by E_r. The process of recombination through incremental steps through the energy band is called **indirect recombination**. The energy loss due to recombination of electron/hole pairs is given up mostly as heat (rather than light). The process of indirect recombination is more complex than direct recombination since carriers that have shifted to a trapping energy level may return to their original energy level without recombining. The rate of recombination may be measured by monitoring the **conductivity** of a sample.

$$\sigma(t) = q[n(t)\mu_n + p(t)\mu_p]$$

σ = (sigma) conductivity [(Ω-cm)$^{-1}$]

q = electron charge 1.6022×10^{-19} [C]

n = concentration of electrons [cm^{-3}]

p = concentration of holes [cm^{-3}]

μ_n = electron mobility [cm^2/V-s] p8.

μ_p = hole mobility [cm^2/V-s]

STEADY STATE ELECTRON AND HOLE CONCENTRATION

The **steady state** condition is when the rate of generation of electron hole pairs equals the recombination rate.

$$n = n_i e^{(F_n - E_i)/kT}$$
$$p = n_i e^{(E_i - F_p)/kT}$$

$$\delta n = \delta p = g_{op}\tau_n$$

n = concentration of electrons [cm^{-3}]

p = concentration of holes [cm^{-3}]

n_i = intrinsic concentration of electrons in the conduction band [cm^{-3}]

E_i = intrinsic energy level [J, eV]

F_n, F_p = quasi Fermi energy level for electrons, holes [J, eV]

δn, δp = excess carrier concentration [cm^{-3}]

g_{op} = optical electron hole pair generation rate [cm^{-3}-s^{-1}]

τ_n = recombination lifetime (applies to minority carriers only) [s]

$\mathbf{F}_B$ POTENTIAL BARRIER HEIGHT

The Schottky potential barrier height for electron injection from the metal into the semiconductor conduction band is

$$q\Phi_B = q\Phi_m - q\chi$$

$q\Phi_B$ = potential barrier height [eV]

$q\Phi_m$ = metal work function [eV]

$q\chi$ = electron affinity [eV]

punch through – An occurrence of **breakdown** below the expected breakdown voltage caused by the transition region extending across the entire width of the layer. This can be a problem in short, lightly-doped regions.

avalanche breakdown – A high-energy electron is driven from the p-region into the transition region by an externally applied voltage. It collides with the lattice, creating an electron-hole pair. The two electrons are swept toward the n-region while the hole is swept toward the p-region under rapid acceleration due to a strong electric field. With sufficient energy, these particles participate in additional lattice collisions within the transition region, causing the creation of additional electron-hole pairs. High reverse-bias voltage and high n-region doping concentrations contribute to the effect, but the **zener (tunneling) effect** takes over at very high doping concentrations.

Zener effect – The n- and p-regions of an abrupt p-n junction must both be heavily doped. The junction is reversed-biased to the extent that the energy bands cross, that is, the energy level of the n-material conduction band is below that of the p-material valence band. These conditions permit electrons to tunnel from E_{vp} to E_{cn} and current flows from n to p as illustrated in the lower diagram at right.

Note that the materials in the diagram at right is **degenerate** type material, meaning that the doping concentration is so high that the Fermi level lies within the valence band of the p-material and within the conduction band of the n-type material.

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Tunneling with forward bias (top) and reverse bias (bottom)
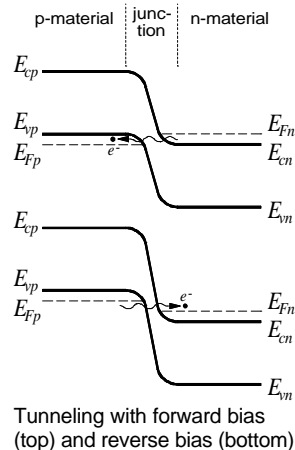
DIODES

Tunnel diode – The tunnel diode operates (in certain regions of its I-V characteristic) by the quantum mechanical tunneling of electrons through the potential barrier of the junction. For reverse current, this is essentially the **zener effect** but requires only a small reverse bias. Applications include high-speed switching and logic circuits.

Photo diodes, solar cells – The junction is operated in the fourth quadrant of its I-V characteristic. The voltage produced is restricted to values less than the contact potential, typically less than 1V. Many compromises are made in solar cell design.

Junction depth in the n material must be less than L_p to allow holes generated near the surface to diffuse to the junction before they recombine.	Many surface contact traces are required to collect current from this thin layer due to its high resistance. But these traces block incoming light.
In order to have a high contact potential V_0, heaving doping is required.	To have long lifetimes, doping must be reduced.
Solar cells are expensive to fabricate and produce only modest amounts of power.	Sunlight may be concentrated to increase output but efficiency is lost. GaAs and related compounds can be used at the higher temperatures.

Photo diodes, photodetectors – The junction is operated in the third quadrant of its I-V characteristic. In this region, current is essentially independent of voltage but proportional to the optical generation rate, making the device useful in measuring illumination levels or converting time-varying optical signals into electrical signals. It is usually desirable for these devices to act quickly, so the transition region W is made large. When an EHP is generated in this region, the electric field quickly sweeps the electron to the n side and the hole to the p side.

Varactor diode – This device employs the voltage-variable capacitance characteristic of a reverse-biased p-n junction. In a graded junction, the relationship between capacitance and voltage is

$$C_j \propto V_r^{-n} \text{ for } V_r \gg V_0$$

For a linearly graded junction, the value of n is ½ The value increases with more abrupt junctions.

Zener diode – The zener diode is a **breakdown diode** designed to break down abruptly at a particular voltage. Although named for the zener or tunneling breakdown mechanism, the **avalanche effect** is actually the more commonly used breakdown mechanism in zener diodes. A common application is voltage regulator.

THE p$^+$-n-p TRANSISTOR

The emitter (p$^+$) is more heavily doped so that the depletion region extends primarily into the base region.
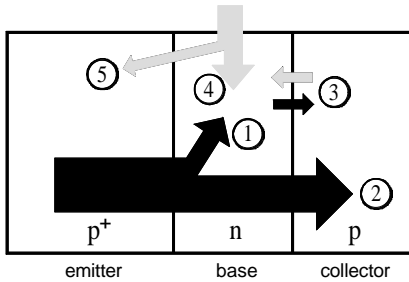
(1) injected holes lost to recombination in the base
(2) holes that fail to be recombined in the base are swept across the reverse-biased collector junction by the electric field
(3) thermally generated electrons and holes are swept across the CB junction by the electric field make up the reverse saturation current of the collector junction
(4) electrons supplied by the base contact recombine with holes from (1), hence (1) and (4) are equal in magnitude
(5) electrons injected across the forward-biased emitter junction; this current is small due to the heavily doped emitter

For a n-p-n transistor, simply reverse the roles of electrons and holes.

NORMAL BIASING IN THE p$^+$-n-p TRANSISTOR

Normal biasing in the p-n-p transistor means that the emitter-base junction is forward-biased and the collector-base junction is reverse-biased. Normal biasing results in reduced width of the emitter-base transition region and increased widthof the collector-base transition region.
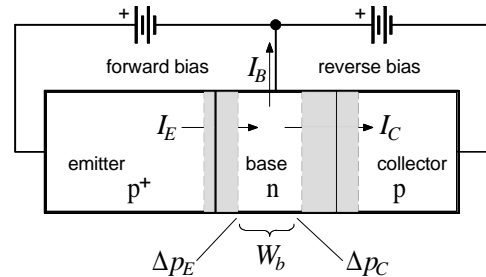
$$\Delta p_E = p_n \left(e^{qV_{EB}/kT} - 1 \right), \quad \Delta p_C = p_n \left(e^{qV_{CB}/kT} - 1 \right)$$

Δp_E, Δp_C = excess hole concentration at the edge of the emitter (Δp_E) and collector (Δp_C) depletion regions [cm^{-3}]

V_{EB}, V_{CB} = emitter to base voltage, normally positive (V_{EB}), collector to base voltage, normally negative (V_{CB}) [V]

W_b = the width of the neutral base region; very important to the speed and efficiency of the transistor; this region is short so that most of the injected holes pass through to the collector (assisted by the electric field at the CB junction) without recombining in the base. [cm]

ENERGY BAND DIAGRAMS FOR THE p$^+$-n-p TRANSISTOR
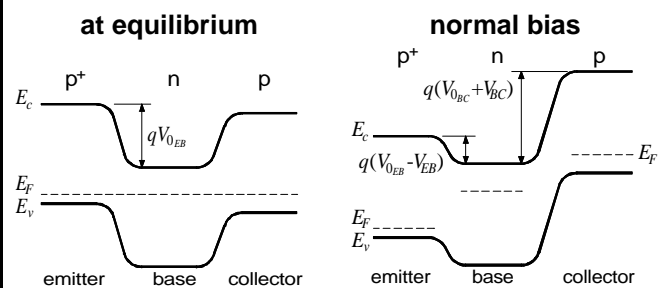
THE DIFFUSION EQUATION AND THE p$^+$-n-p TRANSISTOR

Refer to the previous drawing.

The diffusion equation:
$$\frac{d^2\delta p(x_n)}{dx_n^2} = \frac{\delta p(x_n)}{L_p^2}$$

The diffusion equation solution:
$$\delta p(x_n) = C_1 e^{x_n/L_p} + C_2 e^{-x_n/L_p}$$

Boundary conditions:
$$\delta p(x_n = 0) = C_1 + C_2 = \Delta p_E$$
$$\delta p(x_n = W_b) = C_1 e^{W_b/L_p} + C_2 e^{-W_b/L_p}$$

Solving for C_1 and C_2:
$$C_1 = \frac{\Delta p_C - \Delta p_E e^{-W_b/L_p}}{e^{W_b/L_p} - e^{-W_b/L_p}} \quad C_2 = \frac{\Delta p_E e^{W_b/L_p} - \Delta p_C}{e^{W_b/L_p} - e^{-W_b/L_p}}$$

Δp_E, Δp_C = excess hole concentration at the edge of the emitter (Δp_E) and collector (Δp_C) depletion regions [cm^{-3}]

W_b = the width of the base region not taken up by transition regions [cm]

L_p = diffusion length; the average distance a hole diffuses before recombining [cm]

Dp DIFFUSION IN THE BASE REGION

The following is applicable to a p-n-p transistor; these values are used in other equations. The Δ symbol usually indicates a difference between two values; in this case it would be the difference between the total number of holes and those *not* available for conduction, two values in which we are not interested.

Dp_E = **excess hole concentration** at the boundary of the emitter-base depletion region [cm^{-3}]

Dp_C = **excess hole concentration** at the boundary of the collector-base depletion region ($\Delta p_C = -p_n$ when the collector is strongly reverse-biased) [cm^{-3}]

$$\boxed{\Delta p_E = p_n\left(e^{qV_{EB}/kT} - 1\right)} \quad \boxed{\Delta p_C = p_n\left(e^{qV_{CB}/kT} - 1\right)}$$

p_n = concentration of excess holes in the base region [cm^{-3}]

q = electron charge 1.6022 × 10^{-19} [C]

V_{EB} = emitter-to-base voltage [V]

V_{CB} = collector-to-base voltage [V]

I_E, I_C, I_B TRANSISTOR CURRENTS

Note: ctnh is 1/tanh, csch is 1/sinh.

$$I_{Ep} = qA\frac{D_p}{L_p}\left(\Delta p_E \operatorname{ctnh}\frac{W_b}{L_p} - \Delta p_C \operatorname{csch}\frac{W_b}{L_p}\right)$$

$$I_C = qA\frac{D_p}{L_p}\left(\Delta p_E \operatorname{csch}\frac{W_b}{L_p} - \Delta p_C \operatorname{ctnh}\frac{W_b}{L_p}\right)$$

$$I_B = qA\frac{D_p}{L_p}\left[\left(\Delta p_E + \Delta p_C \tanh\frac{W_b}{2L_p}\right)\right]$$

I_{Ep} = the hole component of the emitter current [A]

I_C = collector current [A]

I_B = base current [A]

q = electron charge 1.6022 × 10^{-19} [C]

A = cross-sectional junction area (uniform for both junctions) [cm^2]

D_p = diffusion coefficient for holes [cm^2/s]

L_p = diffusion length; the average distance a hole diffuses before recombining [cm]

Δp_E = concentration of excess holes at the boundary of the emitter-base depletion region [cm^{-3}]
$\Delta p_E = p_n\left(e^{qV_{EB}/kT} - 1\right)$

Δp_C = concentration of excess holes at the boundary of the collector-base depletion region ($\Delta p_C = -p_n$ when the collector is strongly reverse-biased) [cm^{-3}]
$\Delta p_C = p_n\left(e^{qV_{CB}/kT} - 1\right)$

W_b = width of the base region [cm]

a CURRENT TRANSFER RATIO

The ratio of collector to emitter current, or **common-base current gain**. To make α close to 1, make the base narrow so not many holes recombine in the base.

$$\boxed{\alpha = \frac{I_C}{I_E}} = \frac{b}{a} = \frac{BI_{Ep}}{I_{En} + I_{Ep}} = B\gamma = \frac{\beta}{\beta + 1}$$

$a = (qAD_p/L_p)\operatorname{ctnh}(W_b/L_p)$

$b = (qAD_p/L_p)\operatorname{csch}(W_b/L_p)$

$B = i_c/i_{Ep} = bp_n$, the **base transport factor**, the fraction of injected holes that make it across the base to the collector

I_{Ep} = emitter current due to holes [A]

I_{En} = emitter current due to electrons [A]

γ = emitter injection efficiency

β GAIN FACTOR

The ratio of collector to base current, or **base-to-collector current amplification factor**, or **common-emitter current gain**.

$$\beta = \frac{I_C}{I_B} = \frac{BI_{Ep}}{I_{En} + (1-B)I_{Ep}} = \frac{B\gamma}{1-B\gamma} = \frac{\alpha}{1-\alpha}$$

(see previous for description of variables)

γ EMITTER INJECTION EFFICIENCY

$\gamma \approx 1$ is the emitter current due entirely to hole injection (carrier *injection* always implies minority carriers). To make γ as large as possible, make the emitter heavily doped. The superscripts indicate which side of the emitter-base junction is being referred to.

$$\gamma = \left[1 + \frac{L_p^n n_n \mu_n^p}{L_n^p p_p \mu_p^n} \tanh \frac{W_b}{L_p^n} \right]^{-1} \approx \left[1 + \frac{L_p^n n_n \mu_n^p}{L_n^p p_p \mu_p^n} \right]^{-1}$$

$$= \frac{I_{Ep}}{I_{Ep} + I_{En}}$$

L_p, L_n = diffusion length; the average distance a hole, or electron, diffuses before recombining; the distance at which the excess hole (or electron) distribution is reduced to $1/e$ of its value at the point of injection [cm]

p_p = concentration of holes in the boundary region of the **p** material [cm^{-3}]

n_n = concentration of electrons in the boundary region of the **n** material [cm^{-3}]

W_b = base width in a BJT, measured between the edges of the emitter and collector junction depletion regions [cm]

μ_n = electron mobility [cm^2/V-s] p8.

μ_p = hole mobility [cm^2/V-s]

I_E, I_C EBERS-MOLL EQUATIONS

These **equations for transistor current** allow for asymmetry between the junctions.

$$I_E = I_{EN} + I_{EI} \tag{e1}$$

$$= I_{ES}\left(e^{qV_{EB}/kT} - 1\right) - \alpha_I I_{CS}\left(e^{qV_{CB}/kT} - 1\right) \tag{e2}$$

$$= \alpha_I I_C - I_{EO}\left(e^{qV_{EB}/kT} - 1\right) \tag{e3}$$

$$I_C = I_{CN} + I_{CI} \tag{c1}$$

$$= \alpha_N I_{ES}\left(e^{qV_{EB}/kT} - 1\right) - I_{CS}\left(e^{qV_{CB}/kT} - 1\right) \tag{c2}$$

$$= \alpha_N I_E - I_{CO}\left(e^{qV_{CB}/kT} - 1\right) \tag{c3}$$

To measure the current, use equations e2 and c2. **To get I_{ES},** measure I_E with the collector open and the EB junction reverse-biased. That way, $e^{qV_{EB}/kT}$ is very small. **To get α_I,** make V_{EB} large and negative (reverse-biased), then $\alpha_I = -dI_E/dI_C$. **To get α_N,** make V_{CB} large and negative (reverse-biased), then $\alpha_N = -dI_C/dI_E$. **To get I_{ES},** measure I_C with the emitter open and the CB junction reverse-biased. This information comes from <u>The Physics of Semiconductor Devices</u>, D.A. Fraser, p. 101, and may not be correct—variables were named differently.

I_{EN} = emitter current with $V_{CB} = 0$, i.e. CB shorted [A]

$I_{EI} = \alpha_I I_{CI}$ emitter current, inverted mode (negative value)

I_{ES} = emitter saturation current with $V_{CB} = 0$ [A]

I_{EO} = emitter saturation current with collector open [A]

α_I, α_N = current transfer ratio, inverted/normal modes

I_{CN} = collector current with $V_{CB} = 0$, i.e. CB shorted [A]

I_{CI} = collector current, inverted mode (negative value) [A]

I_{CS} = collector saturation current with $V_{EB} = 0$ [A]

I_{CO} = collector saturation current with emitter open [A]

normal mode means normal biasing
inverted mode means inverted biasing, i.e. reverse-biased emitter and forward-biased collector, making I_{CI} injected current and I_{EI} collected current.

## TRANSISTOR OPERATING MODES

cutoff – the *off* state. I_B is zero or negative.

normal active – emitter junction is forward-biased; collector junction is reverse-biased. operating point is on the load line.

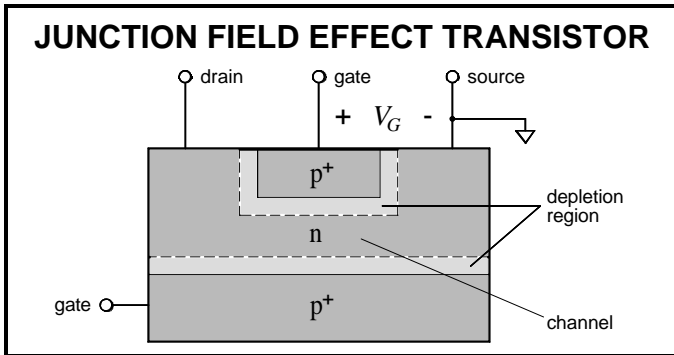**saturation** – collector junction voltage is zero or forward-biased

Excess Hole Distribution in the p-n-p Base

Beginning of Saturation
δ_p
Saturation
x_n
0
W_b
Cutoff
Normal Active
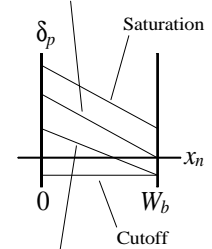
JFETs

JUNCTION FIELD EFFECT TRANSISTOR

V_P PINCH-OFF VOLTAGE

The JFET bias voltage V_{gs} that just causes the depletion regions to meet near the drain, essentially halting further increases in drain current.

$$V_P = \frac{qa^2 N_d}{2\varepsilon}$$

q = electron charge 1.60×10^{-19} [C]

a = channel half-width [m]

N_d = concentration of donors (atoms that may lose an electron) [cm^{-3}]

ε = permittivity of the material [F/cm]

MOSFETs

METAL OXIDE SILICON FIELD EFFECT TRANSISTOR
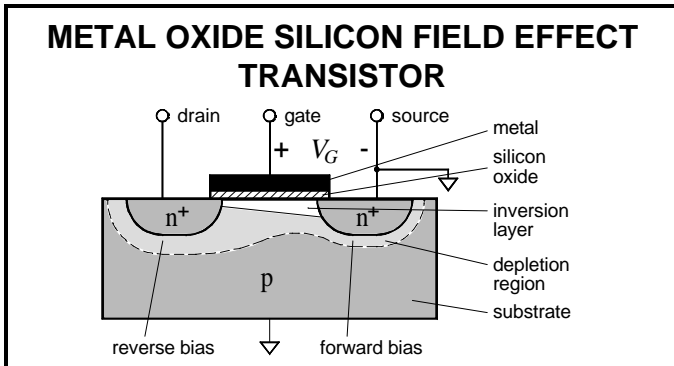
IDEAL MOS AT EQUILIBRIUM

In an ideal MOS, 1) $\Phi_M = \Phi_S$, i.e. there is no band-bending at equilibrium, and 2) there is no charge in the oxide region or at the oxide/semiconductor interface.

E_{Fm} = Fermi level of the metal [eV]

$q\Phi_m$ = modified work function, energy band extending from the metal Fermi level to the conduction band of the silicon oxide [eV]

$q\Phi_s$ = work function, for the semiconductor layer [eV]

$q\phi_F$ = how much lower the Fermi level is than the intrinsic energy level. This value indicates how strongly p-type (n-channel MOSFET) the semiconductor is. [eV] FERMI POTENTIAL p.19.

A break is shown in the energy band of the oxide insulator because its band gap is much greater than that of silicon.
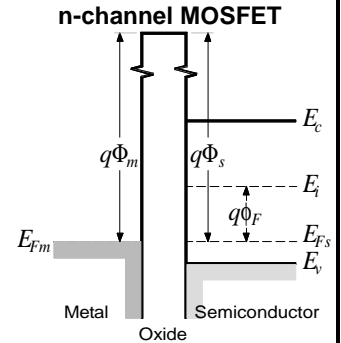
REAL MOS AT EQUILIBRIUM

Trapped charges in the SiO$_2$ layer, as well as charges due to the SiO$_s$/Si interface, produce a layer of positive charge Q_i in the SiO$_2$ material near the substrate boundary. This produces a complementary negative charge Q_d in the substrate. These built-in charges cause **band-bending** to occur in the substrate at equilibrium.
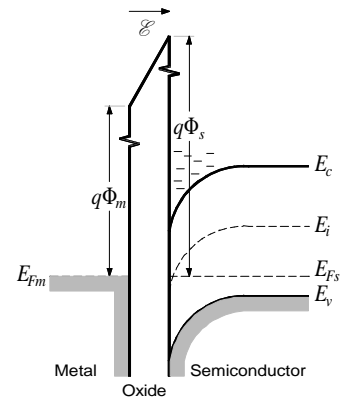
V_G GATE VOLTAGE

The gate voltage can be broken down into components of the voltage drop across the interface, the voltage drop across the depletion region, and the voltage required to achieve flat-band conditions.

$$V_G = V_i + \phi_s + V_{FB}$$

V_T THRESHOLD VOLTAGE

The minimum gate voltage of a MOSFET required to induce the channel. **n-channel MOSFET:** V_T is positive or slightly negative because the first two terms (with their operators) are negative and the last two positive. When an n-channel MOSFET has a negative V_T it means that the channel is made (the MOSFET conducts) until a negative gate voltage is applied. **p-channel MOSFET:** V_T is negative because all terms (with their operators) are negative.

$$V_T = \Phi_{ms} - \frac{Q_i}{C_i} - \frac{Q_d}{C_i} + 2\phi_F \qquad \phi_F = \frac{kT}{q}\ln\frac{N_a}{n_i}$$

Φ_{ms} = metal semiconductor work function potential difference [V]

Q_i = interface charge per unit area [C/cm^2] p.20.

Q_d = depletion region charge per unit area [C/cm^2] p.20.

C_i = interface capacitance per unit area [F/cm^2] p.20.

ϕ_F = Fermi potential; $E_i - E_F$, [V] p.19.

ADJUSTING V_T BY ION IMPLANTATION

The threshold voltage can be made less negative/more positive by implanting boron acceptors just below the Si surface near the gate in both n-channel and p-channel MOSFETs.

$$V_{T(new)} = V_{T(orig.)} + \frac{qF_B}{C_i}$$

F_B = boron doping concentration per unit **area** [cm^{-2}]

C_i = interface capacitance per unit area [F/cm^2] p.20.

V_{FB} FLAT BAND VOLTAGE

The negative voltage required to straighten the energy bands. Trapped charges in the SiO$_2$ layer, as well as charges due to the SiO$_s$/Si interface, produce a layer of positive charge Q_i in the SiO$_2$ material near the substrate boundary (n-channel and p-channel MOSFETs). This produces a complementary negative charge Q_d in the substrate. When the (negative) flat band voltage is applied to the gate, an electric field is set up between the positive charges in the SiO$_2$ layer and the negative charges on the gate, eliminating the negative charges Q_d in the substrate.

Φ_{ms} = metal semiconductor work function voltage [V]

Q_i = effective MOS interface charge per unit area; the naturally occurring charge in the SiO$_2$ layer. Dr. Kwong prefers to call this Q_{ox}. [C/cm^2]

$$V_{FB} = \Phi_{ms} - \frac{Q_i}{C_i}$$

C_i = interface capacitance per unit area [F/cm^2] p.20.

V_i INTERFACE VOLTAGE

The voltage drop across the SiO$_2$ insulator. Dr. Kwong prefers to call this V_{ox}.

$$V_i = \frac{-Q_s d}{\varepsilon_i} = \frac{-Q_s}{C_i}$$

ϕ_s = surface potential [V] p.20.

Q_s = the negative charge density on the semiconductor [C/cm^2]

d = thickness of the insulator [cm]

V_G = gate voltage [V]

C_i = interface capacitance per unit area [F/cm^2] p.20.

ε_i = permittivity of the interface layer, $3.9\varepsilon_0/100$ for SiO$_2$ [F/cm]

$\mathbf{F}_{ms}$ METAL SEMICONDUCTOR WORK FUNCTION [V]

The metal semiconductor work function is the voltage potential between the metal gate and the semiconductor body. Work function voltages vary with the material and the doping concentration. Also, see illustration at REAL MOS AT EQUILIBRIUM p.18.

$$\Phi_{ms} = \Phi_m - \Phi_s$$

Φ_m = work function voltage of the metal gate (high) [V]

Φ_s = work function voltage of the semiconductor (low) [V]
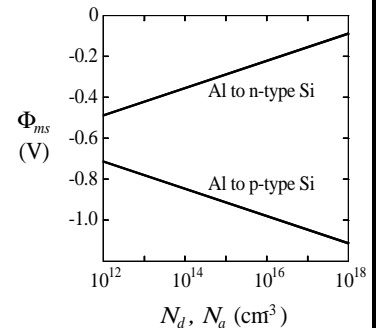
$\mathbf{f}_F$ FERMI POTENTIAL [V]

ϕ_F is the potential between the intrinsic energy level and the Fermi level. It is related to how strongly doped the semiconductor is.
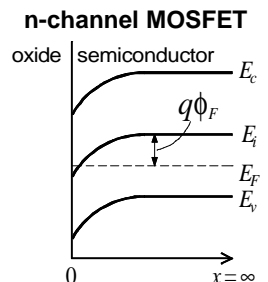
n-channel:

$$\phi_F = \frac{kT}{q}\ln\frac{N_a}{n_i}$$

$$p_0 = n_i e^{q\phi_F/kT}$$

p-channel:

$$\phi_F = -\frac{kT}{q}\ln\frac{N_d}{n_i}$$
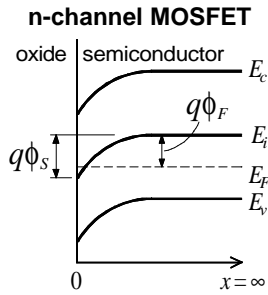
$\mathbf{f}_S$ SURFACE POTENTIAL [V]

ϕ_S is the voltage drop across the depletion region of the semiconductor, i.e. how far the energy bands are bent.

n-channel MOSFET

ϕ_S is a component of the gate voltage:

$$V_G = V_i + \phi_s + V_{FB}$$

at inversion:

$$\phi_S = 2\phi_F$$

oxide | semiconductor
E_c
$q\phi_F$
E_i
E_F
$q\phi_s$
E_v
0 $\quad x = \infty$

Q_n INVERSION CHARGE [C/cm^2]

Q_n is the (variable) charge per unit area resulting from the depletion of holes in the p-type substrate due to positive gate voltage. See MOS CHARGE DENSITY p.21.

$$Q_s = Q_n + Q_d$$

Q_s = surface charge; the total charge density in the substrate [C/cm^2]

Q_d = depletion charge density [C/cm^2]

Q_d DEPLETION CHARGE [C/cm^2]

Q_d is the (fixed) charge density in the depletion region due to Q_i in the interface (SiO$_2$) region. See MOS CHARGE DENSITY p.21. See also FLAT BAND VOLTAGE, p. 19.

n-channel (p-type substrate):

$$Q_d = -qN_aW_m$$
$$= -2\sqrt{\varepsilon_0\varepsilon_r qN_a\phi_F}$$

p-channel (n-type substrate):

$$Q_d = qN_dW_m$$
$$= 2\sqrt{\varepsilon_0\varepsilon_r qN_d|\phi_F|}$$

q = electron charge 1.6022×10^{-19} [C]

N_a = concentration of acceptors (atoms that may gain an electron) [cm^{-3}]

W_m = maximum width of the depletion region [cm] p.21.

$\varepsilon_0\varepsilon_r$ = permittivity of the material [F/cm]

ϕ_F = Fermi potential; $E_i - E_F$, [V] p.19.

Q_i INTERFACE CHARGE [C/cm^2]

Q_i is a (variable) positive charge density in the SiO$_2$ material near the substrate boundary (n-channel and p-channel MOSFETs) composed of trapped charges in the SiO$_2$ layer, as well as charges due to the SiO$_s$/Si interface. With the application of positive gate voltage, the interface charge can be swept out of the insulator region by the electric field. Dr. Kwong prefers to call this Q_{ox}.

Q_s SURFACE CHARGE [C/cm^2]

Q_s is the (variable) total charge density in the depletion region of the semiconductor.

$$Q_s = Q_n + Q_d$$

Q_n = inversion charge density [C/cm^2]

Q_d = depletion charge density [C/cm^2]

I_D DRAIN CURRENT

when V_D is small:

$$I_D = \frac{\overline{\mu}_n ZC_i}{L}\left[(V_G - V_T)V_D - \frac{1}{2}V_D^2\right]$$

more accurately:

$$I_D = \frac{\overline{\mu}_n ZC_i}{L}\left\{\left(V_G - V_{FB} - 2\phi_F - \frac{1}{2}V_D\right)V_D - \frac{2}{3}\frac{\sqrt{2\varepsilon_s qN_a}}{C_i}\left[(V_D + 2\phi_F)^{3/2} - (2\phi_F)^{3/2}\right]\right\}$$

$\overline{\mu}_n$ = surface electron mobility [cm^2/V-s] p8.

Z = depth of the channel [cm]

C_i = interface capacitance per unit area [F/cm^2] p.20.

L = length of the channel [cm]

V_G = gate voltage [V]

V_T = threshhold voltage, the voltage at which the channel begins to conduct [V]

V_D = drain voltage (drain-to-source) [V]

C, C_d, C_i CAPACITANCE [F/cm^2]

The **total capacitance** is the sum of the **interface capacitance** and the **depletion capacitance**.

$$C = A \cdot \frac{\varepsilon}{d} \quad \text{general expression for capacitance [F]}$$

$$C = \frac{C_iC_d}{C_i + C_d} \qquad C_d = \frac{\varepsilon_s}{W} \qquad C_i = \frac{\varepsilon_i}{d}$$

A = area [cm^2]

ε_s = permittivity of the substrate, $11.8\varepsilon_0/100$ for Si [F/cm]

ε_i = permittivity of the interface layer, $3.9\varepsilon_0/100$ for SiO$_2$ [F/cm]

d = separation distance [cm]

C = total capacitance; series capacitance per unit area due to the SiO$_2$ insulator and the depletion capacitance [F/cm^2]

C_i = interface capacitance; (fixed) capacitance per unit area due to the SiO$_2$ insulator [F/cm^2]

C_d = depletion capacitance; capacitance per unit area due to the depletion layer (varies with the gate voltage) [F/cm^2]

W = width of the depletion region [cm] p.21.

W, W_m DEPLETION LAYER WIDTH

The width of the depletion layer within the substrate, including the width of the inversion layer, if present. The width grows with increased gate voltage until reaching maximum at the onset of **strong inversion** (p.21). Further increases in voltage cause stronger inversion rather than more depletion.

$$W = \left[\frac{2\varepsilon_s \phi_s}{qN_a} \right]^{1/2} \qquad W_{max} = 2\left[\frac{\varepsilon_s kT \ln(N_a / n_i)}{q^2 N_a} \right]^{1/2}$$

ε_s = permittivity of the substrate, $11.8\varepsilon_0/100$ for Si [F/cm]

ϕ_s = surface potential [V] p.20.

q = electron charge 1.6022×10^{-19} [C]

n_i = intrinsic concentration of electrons in the conduction band, 1.5×10^{10} for Si [cm^{-3}]

N_a = acceptor concentration in the substrate [cm^{-3}]

STRONG INVERSION

When a positive gate voltage is applied, band-bending takes place. When the gate voltage V_G reaches the **threshold voltage V_T**, the intrinsic energy band has bent down below the Fermi level by a distance equal to the separation of the intrinsic and Fermi energy bands, and the channel area becomes as strongly n-type as the rest of the substrate is p-type, that is, the free electron concentration equals the acceptor doping concentration. This condition is called strong inversion and at this point the channel conducts.

$$n_s = n_i e^{(E_F - E_i)/kT}$$

Strong Inversion:

$$\frac{E_{i\infty} - E_{i0}}{q} = 2\phi_F$$

$$\phi_s = 2\phi_F \Leftrightarrow n_s = N_A$$

n_s = surface concentration of carriers (electrons) in the substrate at the semiconductor/SiO$_2$ transition [cm^{-3}]

n_i = intrinsic concentration of electrons in the conduction band, 1.5×10^{10} for Si [cm^{-3}]
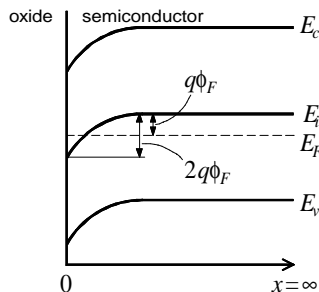
$E_{i\infty}$ = the intrinsic energy level at infinity (far away from the semiconductor/SiO$_2$ interface) [eV]

E_{i0} = the intrinsic energy level at the semiconductor/SiO$_2$ interface [eV]

ϕ_s = surface potential [V] p.20.

ϕ_F = Fermi potential; $E_i - E_F$, [V] p.19.

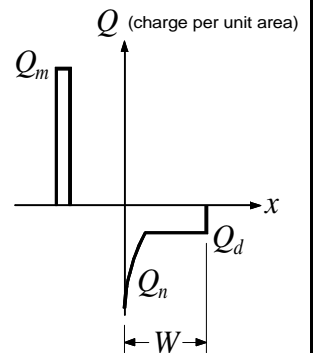N_a = acceptor concentration in the substrate [cm^{-3}]

MOS CHARGE DENSITY

Q_m = gate charge density, due to the positive gate voltage [C/cm^2]

Q_d = depletion region charge density [C/cm^2]

Q_n = inversion charge density, resulting from the depletion of holes due to the positive gate voltage [C/cm^2]
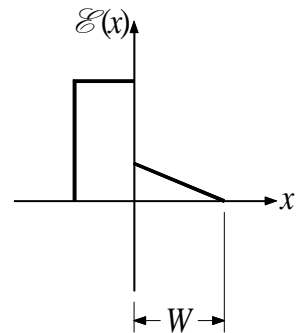

MOS ELECTRIC FIELD

The electric field of an ideal MOSFET in inversion:

W = channel width; corresponds to where band-bending takes place [cm] p.21.

$$W = \left[\frac{2\varepsilon_s \phi_s}{qN_a} \right]^{1/2}$$

MOS ELECTROSTATIC POTENTIAL

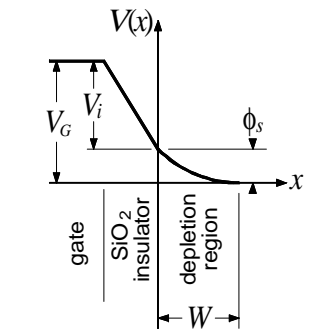$$V_G = V_i + \phi_s \quad \text{(ideal)}$$

$$V_i = \frac{-Q_s d}{\varepsilon_i} = \frac{-Q_s}{C_i}$$

V_G = gate voltage [V]

V_i = interface voltage; voltage drop across the SiO$_2$ interface [V]

ϕ_s = surface potential [V] p.20.

Q_s = surface charge; the total charge density in the substrate [C/cm^2] p.20.


see INTERFACE VOLTAGE p19.

CHANNEL

When the gate voltage V_G reaches the **threshold voltage V_T**, the intrinsic energy band has bent down below the Fermi level by a distance equal to the separation of the intrinsic and Fermi energy bands (strong inversion), and a portion of the the depletion region becomes as strongly n-type as the rest of the substrate is p-type. This area is called the **channel** and is **the conductive path between the drain and source of the MOSFET**.

The change in voltage drop across the depletion region as you move along the x-axis is equal to the drain current times the change in resistance as you move along the x-axis across the width of the channel.

$$dV_x = I_D dR$$
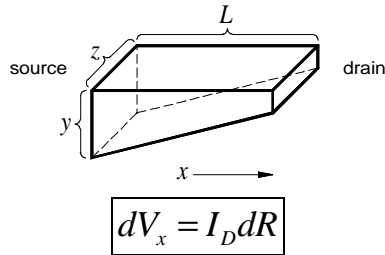
Another way to state this is: the change in current in the depletion region as you move along the x-axis (there isn't any change) is equal to the conductivity of a section Δx times the voltage drop at that point.

$$I_D \, dx = \bar{u}_n z Q_n(x) \, dV_x$$

where: $Q_n(x) = -C_i\left[V_G - V_{FB} - 2\phi_F - V_x - \dfrac{1}{C_i}\sqrt{2q\varepsilon_s N_a(2\phi_F + V_x)} \right]$

We wouldn't want to pass up an opportunity to integrate:

$$\int_0^L I_D \, dx = -\bar{u}_n z \int_{V_D}^0 Q_n(x) \, dV_x$$

The result is the long expression from Drain Current on page 20.

V_x = voltage across the depletion region at some point x along the length of the channel [V] **Point of confusion:** Here, the **length** of the channel (L) is what is referred to elsewhere at the maximum **width** (W_{max}) of the depletion region.

I_D = drain current [A]

dR = the change in the resistivity, which is proportional to the change in area [Ω/m]

$$dR = \frac{1}{\sigma}\frac{dx}{A} = \frac{1}{\sigma}\frac{dx}{zy}$$

σ = (sigma) conductance [Ω^{-1}] $\sigma = qn\mu_n$

A = area [m^2]

$\bar{u}_n$ = electron mobility near the surface [cm^2/V-s] p8.

z = channel depth (see illustration) [m]

V_{FB} = flat-band voltage [V] p19.

N_a = acceptor concentration in the substrate [cm^{-3}]

ϕ_F = Fermi potential; $E_i - E_F$, [V] p.19.

MOSFET DESIGN CONSIDERATIONS

Substrate: Light doping helps guard against breakdown at the drain junction. Doping near the interface by ion implantation adjusts Q_d.

F$_{ms}$ can be reduced by using heavily-doped polycrystalline silicon for the gate material.

Increasing C_i by using a thinner oxide layer or a different material, reduced the magnitude of V_T.

MISCELLANEOUS

NATURAL LOG

$$e^b = x \quad \rightarrow \quad \ln x = b$$

EULER'S EQUATION

$$e^{j\phi} = \cos\phi + j\sin\phi$$

TRIGONOMETRIC IDENTITIES

$$e^{+j\theta} + e^{-j\theta} = 2\cos\theta$$
$$e^{+j\theta} - e^{-j\theta} = j2\sin\theta$$
$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

Ñ NABLA, DEL OR GRAD OPERATOR

Compare the ∇ operation to taking the time derivative. Where $\partial/\partial t$ means to take the derivative with respect to time and introduces a s^{-1} component to the units of the result, the ∇ operation means to take the derivative with respect to distance (in 3 dimensions) and introduces a m^{-1} component to the units of the result. ∇ terms may be called *space derivatives* and an equation which contains the ∇ operator may be called a *vector differential equation*. In other words, $\nabla\mathbf{A}$ is how fast $\mathbf{A}$ changes as you move through space.

in rectangular coordinates:
$$\nabla\mathbf{A} = \hat{x}\frac{\partial A}{\partial x} + \hat{y}\frac{\partial A}{\partial y} + \hat{z}\frac{\partial A}{\partial z}$$

in cylindrical coordinates:
$$\nabla\mathbf{A} = \hat{r}\frac{\partial A}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial A}{\partial \phi} + \hat{z}\frac{\partial A}{\partial z}$$

in spherical coordinates:
$$\nabla\mathbf{A} = \hat{r}\frac{\partial A}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial A}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial A}{\partial \phi}$$

$\tilde{\mathbf{N}}^2$ THE LAPLACIAN

The divergence of a gradient

Laplacian of a **scalar** in **rectangular** coordinates:
$$\nabla^2 A = \frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} + \frac{\partial^2 A}{\partial z^2}$$

Laplacian of a **vector** in **rectangular** coordinates:
$$\nabla^2 \vec{A} = \hat{x}\frac{\partial^2 A_x}{\partial x^2} + \hat{y}\frac{\partial^2 A_y}{\partial y^2} + \hat{z}\frac{\partial^2 A_z}{\partial z^2}$$

In **spherical** and **cylindrical** coordinates:
$$\nabla^2 \mathbf{A} \equiv \nabla(\nabla\cdot\mathbf{A}) - \nabla\times\nabla\times\mathbf{A}$$
$$= \text{grad}(\text{div }\mathbf{A}) - \text{curl}(\text{curl }\mathbf{A})$$

DOT PRODUCT

The dot product is a scalar value.

$\mathbf{A}\cdot\mathbf{B} =$
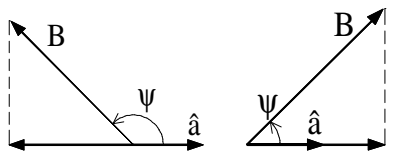$$(\hat{x}A_x + \hat{y}A_y + \hat{z}A_z)\cdot(\hat{x}B_x + \hat{y}B_y + \hat{z}B_z) = A_xB_x + A_yB_y + A_zB_z$$

$$\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos\psi_{AB}$$

$$\hat{x}\cdot\hat{y} = 0, \quad \hat{x}\cdot\hat{x} = 1$$

$$\mathbf{B}\cdot\hat{y} = (\hat{x}B_x + \hat{y}B_y + \hat{z}B_z)\cdot\hat{y} = B_y$$

Projection of **B** along $\hat{a}$:

$$(\mathbf{B}\cdot\hat{a})\hat{a}$$

The dot product is **commutative** and **distributive**:
$$\mathbf{A}\cdot\mathbf{B} = \mathbf{B}\cdot\mathbf{A} \qquad \mathbf{A}\cdot(\mathbf{B}+\mathbf{C}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{C}$$

CROSS PRODUCT

$$\mathbf{A}\times\mathbf{B} = (\hat{x}A_x + \hat{y}A_y + \hat{z}A_z)\times(\hat{x}B_x + \hat{y}B_y + \hat{z}B_z)$$
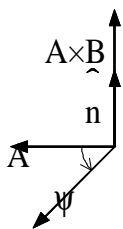$$= \hat{x}(A_yB_z - A_zB_y) + \hat{y}(A_zB_x - A_xB_z) + \hat{z}(A_xB_y - A_yB_x)$$

$$\mathbf{A}\times\mathbf{B} = \hat{n}|\mathbf{A}||\mathbf{B}|\sin\psi_{AB}$$

where $\hat{n}$ is the unit vector normal to both **A** and **B** (thumb of right-hand rule).

$$\mathbf{B}\times\mathbf{A} = -\mathbf{A}\times\mathbf{B}$$

The cross product is **distributive:**
$$\mathbf{A}\times(\mathbf{B}+\mathbf{C}) = \mathbf{A}\times\mathbf{B} + \mathbf{A}\times\mathbf{C}$$

CONSTANTS

Avogadro's number [molecules/mole]	$N_A = 6.02\times10^{23}$
Boltzmann's constant	$k = 1.38\times10^{-23}$ J/K
or	$K = 8.62\times10^{-5}$ eV/K
Elementary charge	$q = 1.6022\times10^{-19}$ C
Electron mass	$m_0 = 9.11\times10^{-31}$ kg
Permittivity of free space	$\varepsilon_0 = 8.85\times10^{-14}$ F/cm
	$\varepsilon_0 = 8.85\times10^{-12}$ F/m
Planck's constant	$h = 6.63\times10^{-34}$ J-s
	$= 4.14\times10^{-15}$ eV-s
	also, $\hbar = h/(2\pi)$
Rydberg constant	$R = 109,678$ cm^{-1}
kT @ room temperature	$kT = 0.0259$ eV
Speed of light	$c = 2.998\times10^{10}$ cm/s
1 Å (angstrom)	10^{-8} cm = 10^{-10} M
1 μm (micron)	10^{-4} cm

1 nm = 10Å = 10^{-7} cm

1 eV = 1.6022 × 10^{-19} J   1 W = 1 J/S = 1 VA

1 V = 1 J/C   1 N/C = 1 V/m   1 J = 1 N· m = 1 C· V

EXPANSIONS FOR HYPERBOLIC FUNCTIONS

$$\text{sech } y = 1 - \frac{y^2}{2} + \frac{5y^4}{24} - \cdots \qquad \text{csch } y = \frac{1}{y} - \frac{y}{6} + \frac{7y^3}{360} - \cdots$$

$$\text{ctnh } y = \frac{1}{y} + \frac{y}{3} - \frac{y^3}{45} + \cdots \qquad \text{tanh } y = y - \frac{y^3}{3} + \cdots$$