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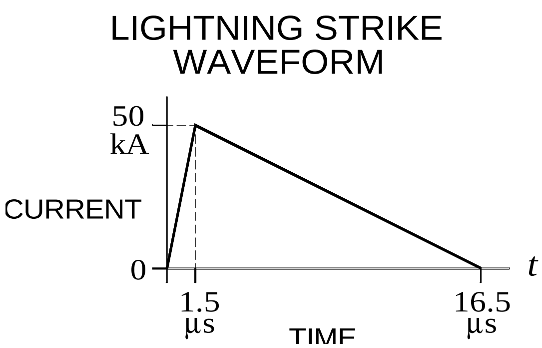
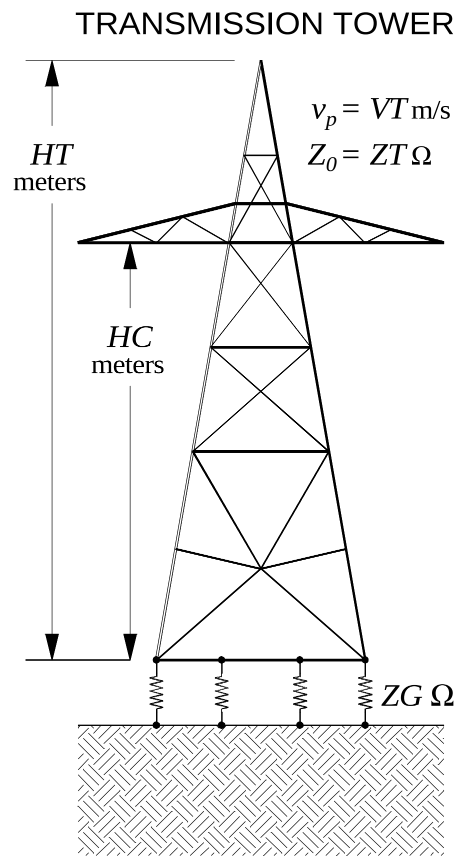
EE368 Electrical Power Transmission and Distribution
Homework 9, 4/21/99

Problem:

A single transmission tower with no conductors, or ground wires, has a height of HT=35 meters, a crossarm at HC=22 m, a surge impedance of ZT=80 Ω, a velocity of propagation of VT=250 m/μs, and 4 legs, each with a grounding resistance of ZG=20 Ω. A 1.5 x 50 (1.5 μs rise time, 50 kA peak, 15 μs fall time) lightning bolt hits the top of the tower.

Compute and plot the voltage with respect to ground at the top of the tower for $0 \le t \le 2$ μs.





Program Overview:

In addition to the given values, a resolution value is selected (1000). This value determines the number of points that are calculated and plotted (Resolution + 1) and the time interval between each point (Resolution / Range).

The lightning strike waveform begins on the *t*-axis, makes a straight-line excursion away from the axis (above or below, depending on the reflection coefficients) followed by a second straight-line excursion terminating at the *t*-axis. The program calculates the slopes of the two excursions based on the initial strike current. For the lightning strike we have two functions of the form $y = mt + b$, where $b = 0$ for the first function and b = 50kA for the second. The values of m_1, m_2, b_1 and b_2 remain constant while the functions are scaled and reflected across the *t*-axis using impedance and reflection factors, and offset with respect to time by subtracting

appropriate offset values from *t*. So the function takes on the form
$y = \{[m(t - \text{offsets}) + b] \times \text{scale factors}\}$.

A matrix called Time is created and filled with the time values to be used in calculations and plotting. Time has 1 row and Resolution + 1 columns. A matrix called VGtop is created and filled with zeros. VGtop has a row for each reflection that will occur during the Range period and has Resolution + 1 columns.

The waveform of each reflection is sampled mathematically at each value from the Time matrix which falls within the domain of the strike function. Results, in terms of voltage, are placed in the VGtop matrix. For Time values falling outside the domain of the strike function, corresponding values in the VGtop matrix are left at zero.

Since, in this assignment, we are concerned with the voltage at the top of the tower, the first reflection will present its effect at $t = 2\times$the propagation period, simultaneous with the second reflection. This is hard-coded into this program, but can be modified to give the voltage at any point along the length of the tower since each reflection is calculated separately.

A third matrix, VG, is created and filled with the sums of the columns of VGtop. Matrix VG has 1 row and has Resolution + 1 columns. We now have the data to plot each reflection as well as the combined result.

Matlab Program:

```
% *************  GIVEN VALUES  ****************************************

format short                 % Instruction to display 5-digit results
HT = 35                      % Height of Tower [m]
HC = 22                      % Height of Crossarm [m]
ZT = 80                      % Surge Impedance [W]
VT = 2.5e8;                  % Velocity of Propagation [m/s]
Legs = 4;                    % Number of Tower Legs
ZG = 20;                     % Leg Grounding Resistance [W]
RiseTime = 1.5e-6;           % Lightning Strike Rise Time
PeakAmps = 50e3;             % Lightning Strike Peak Amps
FallTime = 15e-6;            % Lighening Strike Fall Time
Range = 2e-6;                % The total range (time) to be plotted
Resolution = 1000            % The number of points to be plotted
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% **********  ADDITIONAL VARIABLES  ***********************************

VGtop = [];                  % Tower Top Voltage to Ground [V]
Time = [];                   % Lightning Strike Rise Time [s]
ZL = 0;                      % Tower to Ground Resistance [W]
PBv = 0;                     % Reflection Coefficient of Voltage at Base
PTv = 1;                     % Reflection Coefficient of Voltage at Top
Slope1 = 0;                  % Slope of the rising Current Curve
Slope2 = 0;                  % Slope of the falling Current Curve
ReflCnt = 0;                 % Which reflection we are working on
ReflTime = 0;                % Reflection period [s]
TotRefCoef = 1;              % Combination of reflection coefficients

% *********************  INITIAL CALCULATIONS  ************************

ZL = ZG/Legs                 % Calculate Tower to Ground Resistance
PBv = (ZL-ZT)/(ZL+ZT)        % Calculate Refl. Coef. of Voltage at Base
Slope1 = PeakAmps/RiseTime   % Slope of the rising Current Curve
Slope2 = -PeakAmps/FallTime  % Slope of the falling Current Curve
ReflTime = HT/VT             % Reflection period

% ***************  CREATE TIME AND VOLTAGE MATRICES  ******************

% This routine creates a set of time values to be used in calculating
% voltages.  The number of elements and range of their values determines
% the resolution and range of the plot.  The Voltage matrix is filled
% with an equal number of zeros.

for Cnt = 0:Range/Resolution:Range
  Time = [Time Cnt];
end
  VGtop = zeros(ceil(Range/ReflTime+1),Resolution+1);

% ********************  CALCULATE VOLTAGE MATRIX  *********************

% This routine calculates the voltage at the top of the tower due to
% the initial lightning strike for each value in the Time matrix
% and stores the result in the 1st row of matrix VGtop.  Then the
% tower top voltage is recalculated for each reflection and these
% values are stored in separate rows in the matrix VGtop.

StartTime = 0;               % The time at which calculations begin
LocCnt = 1;                  % Where it goes in the matrix
% LocCnt is used to determine what values of Time are used
% in calculating the value of the voltage in the wave.  So the
% initial value of LocCnt must be set so that the corresponding
% value in the Time matrix falls just within the waveform and
% calculations must stop before the Time value corresponding
% to LocCnt falls past the waveform so that invalid values are
% not incorporated into the Voltage matrix.
```

```
% The following loop repeats after every two reflections.  It
% contains nested loops that calculate rising and falling segments
% of the even and odd reflections.  Nested loops terminate when
% either the time value exceeds the boundary of the waveform
% segment (if statement) or the time value exceeds the
% specified range (while statement).
while ReflCnt*ReflTime <= Range

  % The following loop calculates the rising slope of the forward-
  % traveling wave.  First, factor in the top reflection coefficient.
  TotRefCoef = TotRefCoef*PTv;
  while LocCnt <= 1+Resolution
    if Time(LocCnt) > ReflCnt*ReflTime+RiseTime,break,end
    VGtop(ReflCnt+1,LocCnt) = Slope1*(Time(LocCnt)-
       ReflCnt*ReflTime)*ZT*TotRefCoef;
    LocCnt = LocCnt + 1;
  end

  % The following loop calculates the falling slope of the forward-
  % traveling wave.
  while LocCnt <= 1+Resolution
    if Time(LocCnt) > ReflCnt*ReflTime+RiseTime+FallTime,break,end
    VGtop(ReflCnt+1,LocCnt) = (Slope2*(Time(LocCnt)-RiseTime-
       ReflCnt*ReflTime)+PeakAmps)*ZT*TotRefCoef;
    LocCnt = LocCnt + 1;
  end
  ReflCnt = ReflCnt+1;                  % Count the reflection

  % An additional ReflTime is added below to account for the delay
  % before the wave reflected from the base is "felt" at the top
  % of the tower.  The factor Resolution/Range converts the elapsed
  % time to units of LocCnt.  1 LocCnt = 1 Range/Resolution.
  % ceil() rounds up to the next integer.  The wave reflected at
  % the top will appear simultaneously but will be added later in
  % the program.
  LocCnt = ceil((ReflCnt*ReflTime+ReflTime)*Resolution/Range);

  % The following loop calculates the rising slope of reverse-
  % traveling wave.  First, factor in the tower base reflection
  % coefficient.
  TotRefCoef = TotRefCoef*PBv;
  while LocCnt <= 1+Resolution
    if Time(LocCnt) > ReflCnt*ReflTime+ReflTime+RiseTime,break,end
    VGtop(ReflCnt+1,LocCnt) = Slope1*(Time(LocCnt)-ReflCnt*ReflTime-
       ReflTime)*ZT*TotRefCoef;
    LocCnt = LocCnt + 1;
  end
```

```
  % The following loop calculates the falling slope of reverse-
  % traveling wave.
  while LocCnt <= 1+Resolution
    if Time(LocCnt) >
       ReflCnt*ReflTime+ReflTime+RiseTime+FallTime,break,end
    VGtop(ReflCnt+1,LocCnt) = (Slope2*(Time(LocCnt)-RiseTime-
       ReflCnt*ReflTime-ReflTime)+PeakAmps)*ZT*TotRefCoef;
    LocCnt = LocCnt + 1;
  end

  ReflCnt = ReflCnt+1;                  % Count the reflection
  % LocCnt rewinds to the previous value in order to calculate the
  % wave reflected from the tower top. The factor Resolution/Range
  % converts the elapsed time to units of LocCnt.
  % 1 LocCnt = 1 Range/Resolution.
  LocCnt = ceil(ReflCnt*ReflTime*Resolution/Range);
end

% **************  CREATE PLOTS OF ALL REFLECTIONS  ********************

Width = 1200; Height = 900;
H = figure('Position',[20 20 Width Height],'Color',[1 .6 .9])

hold on
Cnt = 1;
for Cnt = 1:1:ReflCnt
  plot(Time*10e5,VGtop(Cnt,:)*10e-7,'k-');
end
hold off
grid on
% Label the Plot:
set(gca,'FontSize',16,'Xcolor','k','Ycolor','k')
set(gca,'xtick',[0:.1:2])
title('Voltage Waves at the Top of a Transmission Tower due to a 1.5x50
 Lightning Strike','FontSize',18,'Color',[0 0 0])
xlabel('Time in Microseconds','FontSize',16, 'Color',[0 0 0])
ylabel('Voltage in Megavolts','FontSize',16, 'Color',[0 0 0])
```

```
% *************  CREATE OVERALL VOLTAGE VS TIME PLOT  *****************

for Cnt = 1:1:size(VGtop,2)      % Sum the columns
  VG(Cnt) = sum(VGtop(:,Cnt));
end
Width = 1200; Height = 900;
H = figure('Position',[20 20 Width Height],'Color',[1 .6 .9])

plot(Time*10e5,VG*10e-4,'k-');
axis([0 2 -300 900])
grid on
% Label the Plot:
set(gca,'FontSize',16,'Xcolor','k','Ycolor','k')
set(gca,'xtick',[0:.1:2])
set(gca,'ytick',[-300:50:900])
title('Voltage at the Top of a Transmission Tower due to a 1.5x50
 Lightning Strike','FontSize',18,'Color',[0 0 0])
xlabel('Time in Microseconds','FontSize',16, 'Color',[0 0 0])   % label
 the x-axis
ylabel('Voltage in Kilovolts','FontSize',16, 'Color',[0 0 0])

% ********************  CREATE A TEST PLOT  ***************************

% Test Plot of initial wave and 1st reflection (for debugging).
Width = 1200; Height = 900;
H = figure('Position',[20 20 Width Height],'Color',[1 .6 .9])
hold on
plot(Time,VGtop(1,:),'k-'); plot(Time,VGtop(2,:),'k-');
hold off; grid on
set(gca,'FontSize',16,'Xcolor','k','Ycolor','k')
title('Test Plot','FontSize',18,'Color',[0 0 0])
xlabel('Time in Seconds','FontSize',16, 'Color',[0 0 0])
ylabel('Voltage in Volts','FontSize',16, 'Color',[0 0 0])

% ************************  END OF PROGRAM  ***************************
```

Formulas Used:

$$I = mt + b$$

$$m_R = \frac{I_P}{t_R} \qquad m_F = \frac{I_P}{-t_F}$$

$$V_R = m_R\left(t - N_R \frac{h}{v_p}\right) Z_0 \rho_{tot}$$

$$V_F = \left[m_F\left(t - t_R - N_R \frac{h}{v_p}\right) + I_P\right] Z_0 \rho_{tot}$$

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0}$$

I = current [A]

I_P = peak current [A]

m = slope of current plot

m_R, m_F = slopes of the rising and falling currents

t = time [s]

t_R, t_F = time to rise, time to fall [s]

b = y-intercept of current plot [A]

V_R, V_F = Rising voltage, falling voltage [V]

N_R = Number of reflections

h = Height of tower [m]

v_p = Velocity of propagation [m/s]

Z_0 = Surge impedance of the tower [Ω]

Z_L = Load impedance (tower base) [Ω]

r, r_{tot} = Reflection coefficient, total reflection coefficients (all multiplied together)

Initial Program Output:

HT = 35

HC = 22

ZT = 80

Resolution = 1000

ZL = 5

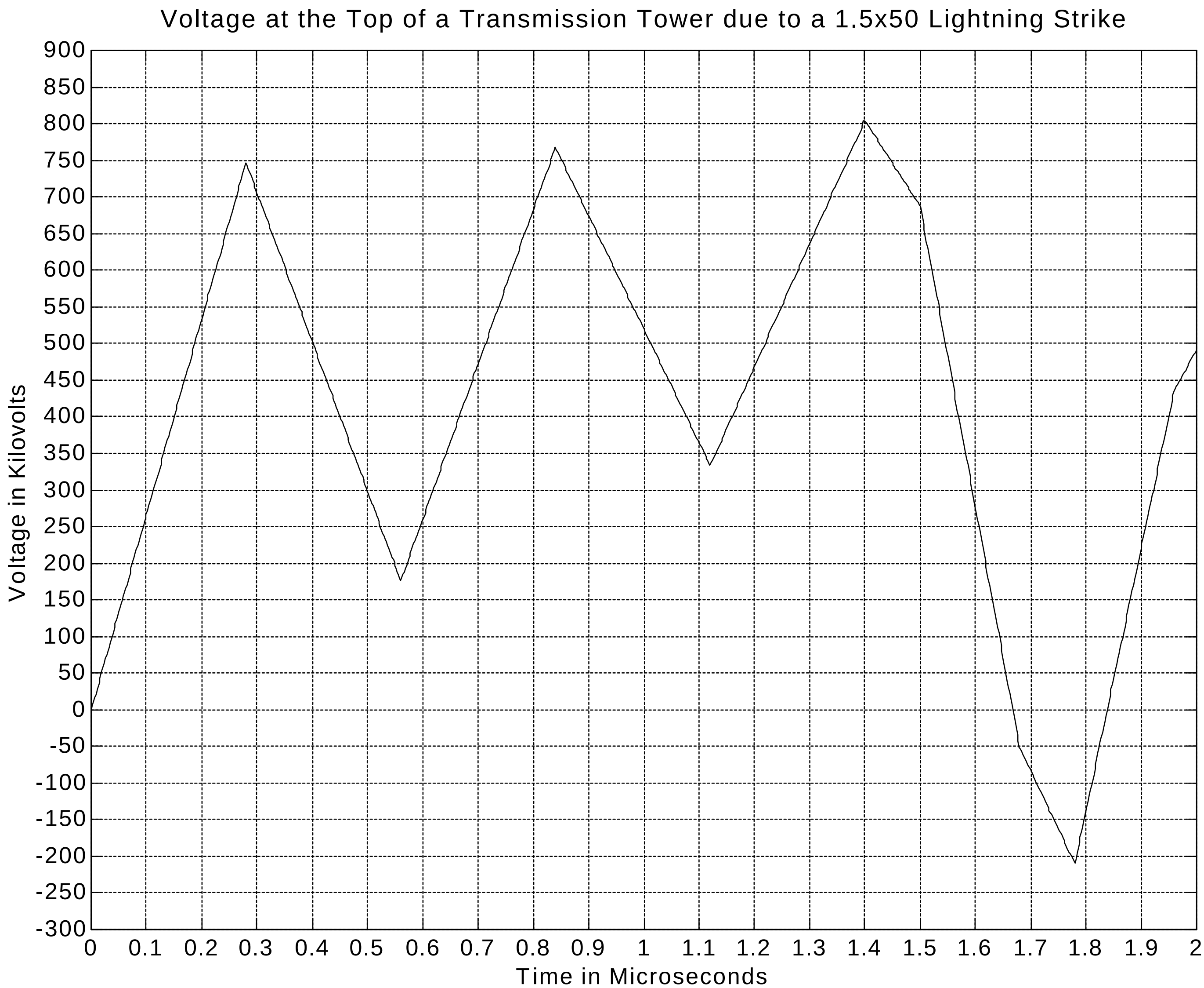
PBv = -0.8824

Slope1 = 3.3333e+010

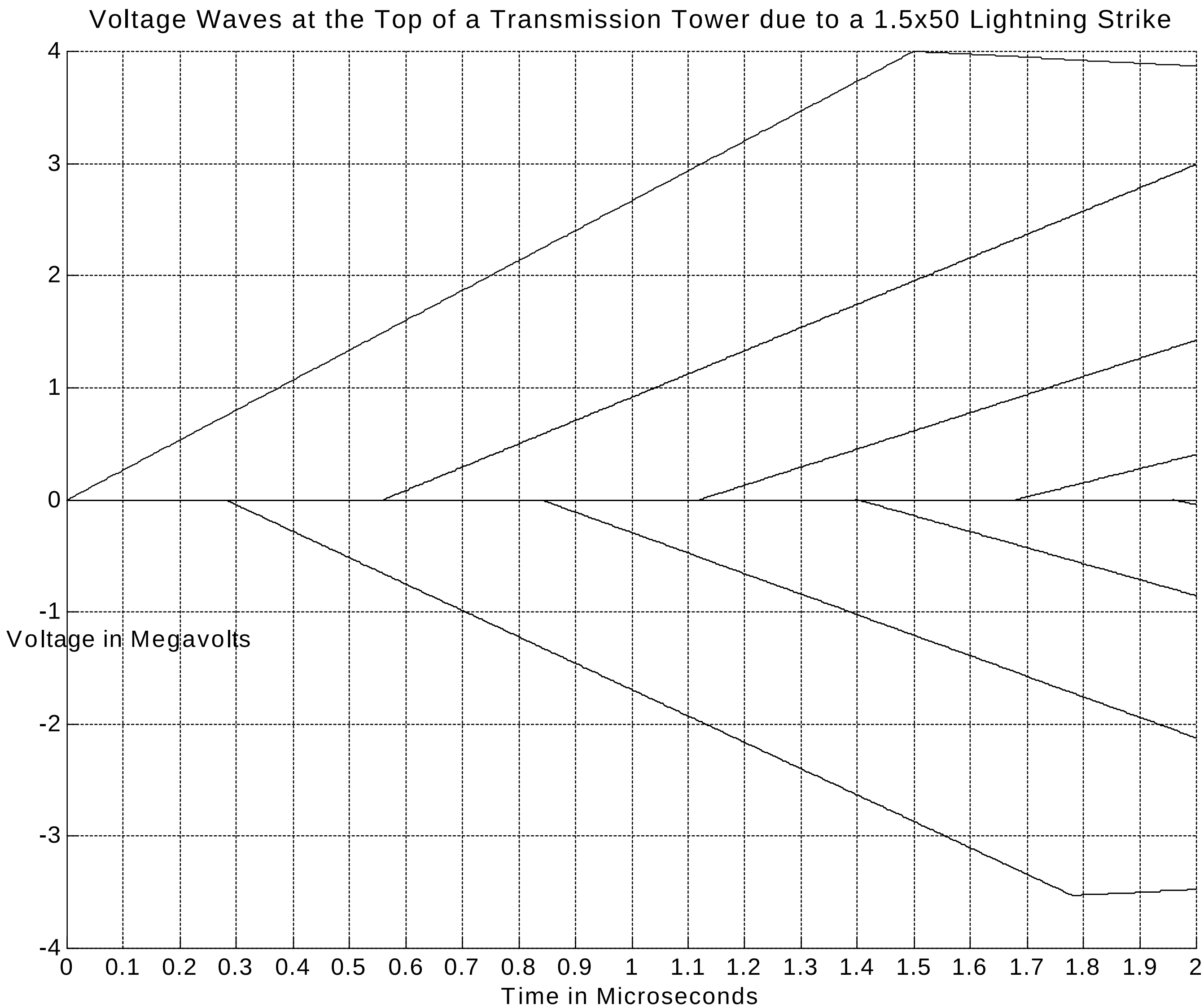
Slope2 = -3.3333e+009

ReflTime = 1.4000e-007

Plot of Voltage at the Top of the Tower:



Plot of Voltage Waves at the Top of the Tower:



Note that each plot shown except for the initial wave (the initial wave intersects the origin) represents 2 plots superimposed, an upward traveling wave and its equal downward reflection.