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EE368 Electrical Power Transmission and Distribution
Homework 1
1/29/99

Problem:

A single 795ACSR conductor is 10 meters above the earth. Assume the radius is 1 centimeter. R = 0, G = 0 (lossless line). $V_R = 100\ kV_{RMS} \angle 0°$, S = 10 MVA @ pf = 0.90 lagging, f = 60 Hz.

Plot the voltage magnitude and current magnitude over a distance of 5000 km.

Matlab Program:

```
% *************  GIVEN VALUES  ***********************************

format short                % 5-digit results
PwrOut = 10000000;          % Output Power
pf = .9;                    % Power Factor
OutputVoltage = 100000;     % (phase angle 0)
Frequency = 60;             % Frequency 60 [Hz]
HeightAboveGround = 10;     % Height of transmission line [m]
Radius = .01;               % radius of the conductor [m]
Length = 5000000;           % length of line [m] (5000 kilometers)
Epsilon = 8.85e-12;         % Permittivity of free space constant [F/m]
Mu = 4*pi*10^-7             % constant [T·m/A]
Omega = Frequency*2*pi      % Frequency converted to radians
R = 0;                      % Resistance per meter [Ohms/m]
G = 0;                      % Conductance [V/m]

% **************  CALCULATIONS  *********************************

%Output Power – Use to obtain Output Current
RadianPwrOut = -acos(pf)          %Phase angle
DegreesPowerOutput = rad2deg(RadianPwrOut)
Real=PwrOut*cos(RadianPwrOut);
Cplx=PwrOut*sin(RadianPwrOut);
% Convert two real variables to a complex number
VectorPwrOut = j*Cplx+Real;
OutputCurrent = VectorPwrOut/OutputVoltage

% Capacitance per meter of length
C = (Epsilon*2*pi)/log(2*HeightAboveGround/Radius)

% Inductance per meter of length
L = (Mu/(2*pi))*log(2*HeightAboveGround/Radius)

% Surge impedance
z0 = ((j*Omega*L)/(j*Omega*C))^(1/2)
```

```
% Attenuation constant, Alpha
Alpha = (R/2)*(C/L)^(1/2) + (G/2)*(L/C)^(1/2)

% Phase constant, Beta
Beta = Omega * (L*C)^(1/2)

% Propagation constant, Gamma
Gama = Alpha + j*Beta

% Wavelength, Lambda [m]
Lambda = 2*pi/(Omega*(L*C)^(1/2))

% ************  CREATE PLOTS FOR VOLTAGE MAGNITUDE,  ****************
%               CURRENT MAGNITUDE, AND PHASE

% CP2MP() returns the magnitude of a complex number
% phase() returns the phase of a complex number in radians

newplot
x = -5000000 : 1000 : 0; axis([-5000000 0 -180 100000])
V = (OutputVoltage*cosh(Gama*(-x))+OutputCurrent*z0*sinh(Gama*(-x)));
plot(x,CP2MP(V),'k-'); grid on           % Plot voltage
xlabel('Distance in Meters')        % label the x-axis
ylabel('Volts')                     % label the y-axis

figure; newplot;                     %opens new window for second plot
x = -5000000 : 1000 : 0; axis([-5000000 0 -180 250])
I = (OutputVoltage*sinh(Gama*(-x))/z0+OutputCurrent*cosh(Gama*-(x)));
plot(x,CP2MP(I),'k-'); grid on           % Plot current
xlabel('Distance in Meters')        % label the x-axis
ylabel('Current in Amps')           % label the y-axis

figure; newplot;                    % While we're at it, plot phase
x = -5000000 : 80000 : 0; axis([-5000000 0 -180 180])
V = (OutputVoltage*cosh(Gama*(-x))+OutputCurrent*z0*sinh(Gama*(-x)));
I = (OutputVoltage*sinh(Gama*(-x))/z0+OutputCurrent*cosh(Gama*(-x)));
plot(x,360+phase(I)*180/pi,'k+',x,360+phase(V)*180/pi,'k:'); grid on
xlabel('Distance in Meters       ++++Amps     -----Volts')  % x-axis
ylabel('Phase in Degrees')          % label the y-axis

% **********  CREATE MATRIX OF VALUES FOR DATA SHEET  **************

x = 0;                               % Counter (meters)
Kilometers = []; VoltageMagnitude = []; VoltagePhaseAngle = [];
CurrentMagnitude = []; CurrentPhaseAngle = [];
while x > -Length-1
Vcomplex = OutputVoltage*cosh(Gama*(-x))+OutputCurrent*z0*sinh(Gama*(-x));
Icomplex = OutputVoltage*sinh(Gama*(-x))/z0+OutputCurrent*cosh(Gama*(-x));
Kilometers = [Kilometers;x/1000];
VoltageMagnitude = [VoltageMagnitude;CP2MP(Vcomplex)/1000];   % [KV]
VoltagePhaseAngle = [VoltagePhaseAngle;phase(Vcomplex)*180/pi];
CurrentMagnitude = [CurrentMagnitude;CP2MP(Icomplex)];        % [A]
CurrentPhaseAngle = [CurrentPhaseAngle;phase(Icomplex)*180/pi];
x = x – 100000;                      % Decrement the counter
end
format bank                          % Results to 2 decimal places
Output = [Kilometers VoltageMagnitude VoltagePhaseAngle CurrentMagnitude
CurrentPhaseAngle]
```

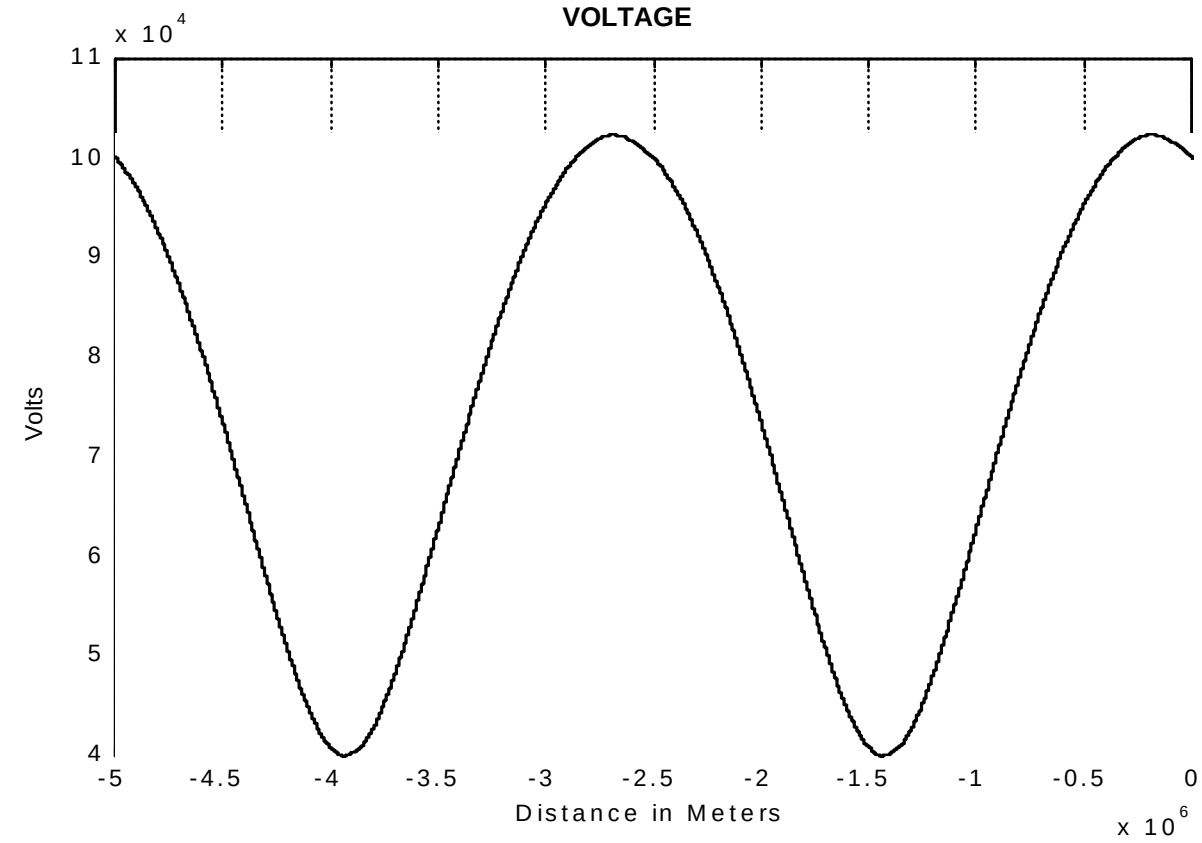
Formulas Used:

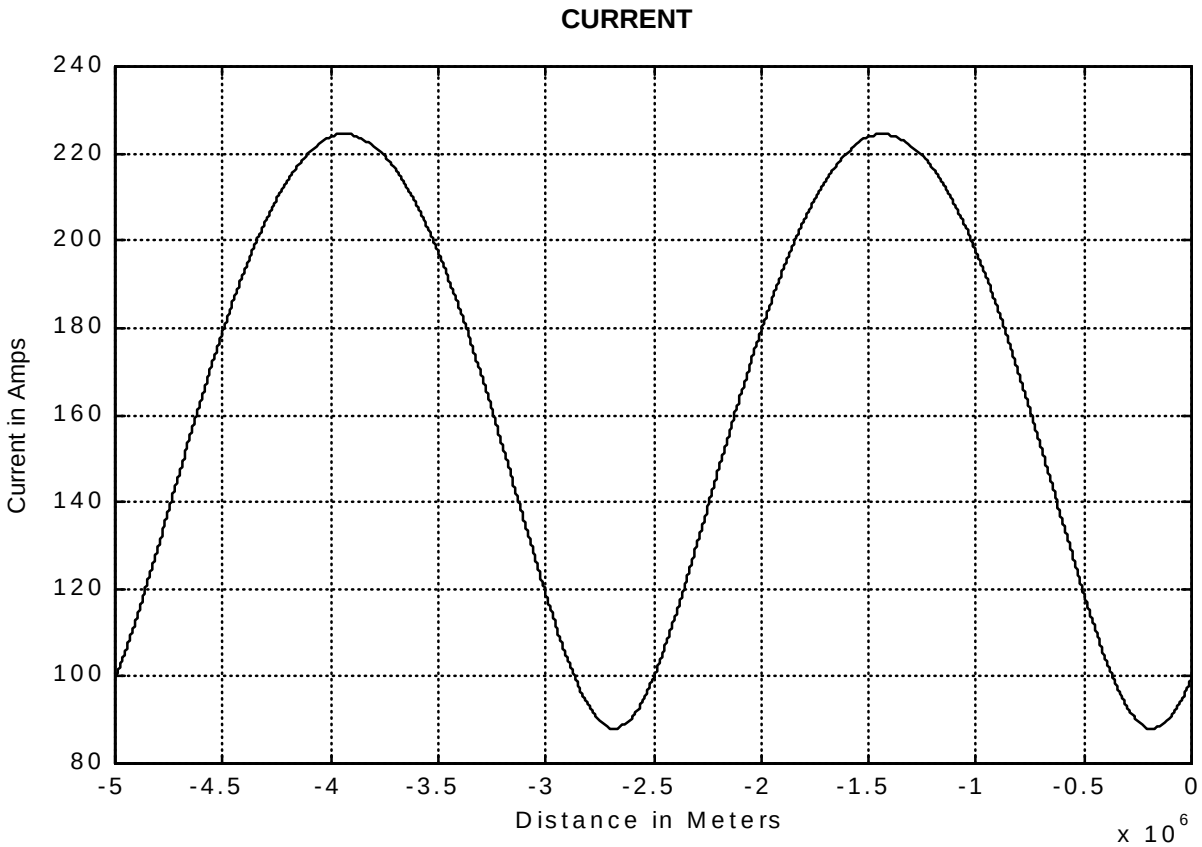
$$C = \frac{2\pi\varepsilon_0}{\ln\frac{2h}{r}}$$	ε_0 = Permittivity of free space 8.85×10^{-12} [F/m]
$$z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$	h = height of transmission line [m]
$$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}$$	r = radius of the conductor [m]
$$\beta = \omega\sqrt{LC}$$	μ = (mu) constant $4\pi\times10^{-7}$ [T·m/A]
$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$	α = attenuation constant
$$\mathbf{V}_S = \mathbf{V}_R \cosh\gamma d + z_0\mathbf{I}_R \sinh\gamma d$$	β = phase constant [rad./m]
$$\mathbf{I}_S = \frac{\mathbf{V}_R}{z_0}\sinh\gamma d + \mathbf{I}_R \cosh\gamma d$$	γ = (gamma) propagation constant
	z_0 = surge impedance (has nothing to do with resistance) [Ω]
	R = resistance [Ω/m]
	L = inductance [H/m]
	G = conductance [v/m]
	C = capacitance [F/m]
	d = length of the line [m]
	$\mathbf{V}_S$ = voltage (complex) at the sending end [v]
	$\mathbf{V}_R$ = voltage (complex) at the receiving end [v]

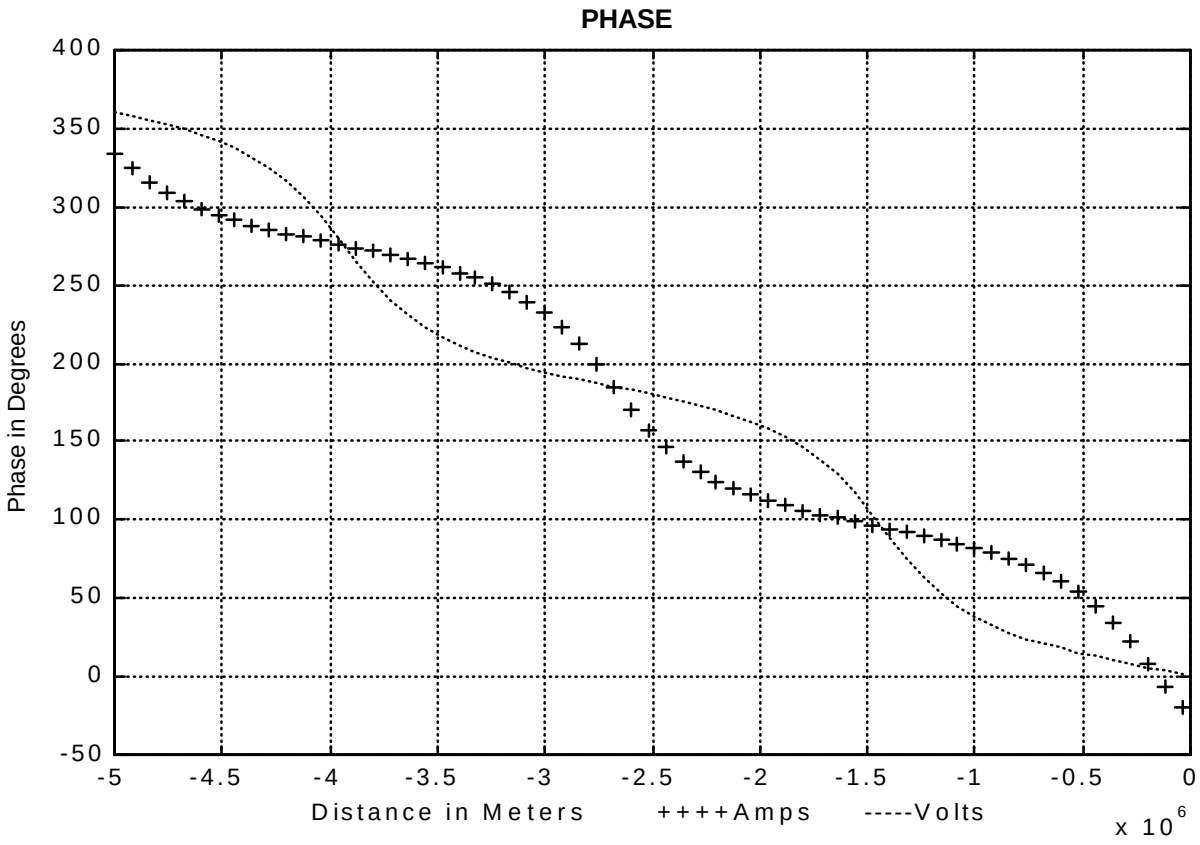
Program Output:

Mu = 1.2566e-006
Omega = 376.9911
RadianPwrOut = -0.4510
DegreesPowerOutput = -25.8419
OutputCurrent = 90.0000 +43.5890i
C = 7.3157e-012
L = 1.5202e-006
z0 = 455.8465
Alpha = 0
Beta = 1.2572e-006
Gama = 0 +1.2572e-006i
Lambda = 4.9977e+006

Plots:


VOLTAGE
x 10^4
11
10
9
8
7
6
5
4
Volts
-5
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
Distance in Meters
x 10^6


CURRENT
240
220
200
180
160
140
120
100
80
Current in Amps
-5
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
Distance in Meters
x 10^6


PHASE
400
350
300
250
200
150
100
50
0
-50
Phase in Degrees
-5
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0
Distance in Meters
++++Amps
-----Volts
x 10^6

Voltage and Current Data:

KILOMETERS	VOLTAGE (KW)	V. PHASE (°)	CURRENT (AMPS)	CURRENT (°)
0	100.00	0	100.00	-25.84
-100.00	101.83	2.90	90.67	-10.00
-200.00	102.31	5.73	88.04	8.07
-300.00	101.42	8.57	92.86	25.70
-400.00	99.19	11.50	103.82	40.58
-500.00	95.66	14.61	118.68	52.17
-600.00	90.93	18.00	135.40	61.03
-700.00	85.12	21.81	152.50	67.91
-800.00	78.40	26.23	168.95	73.43
-900.00	71.00	31.53	184.03	77.99
-1000.00	63.23	38.11	197.19	81.91
-1100.00	55.53	46.54	208.05	85.37
-1200.00	48.53	57.54	216.31	88.52
-1300.00	43.10	71.78	221.77	91.48
-1400.00	40.30	89.01	224.30	94.33
-1500.00	40.84	107.22	223.83	97.16
-1600.00	44.57	123.64	220.37	100.03
-1700.00	50.60	136.83	214.02	103.04
-1800.00	57.90	146.94	204.92	106.28
-1900.00	65.68	154.72	193.30	109.86
-2000.00	73.37	160.84	179.49	113.95
-2100.00	80.59	165.83	163.92	118.78
-2200.00	87.04	170.04	147.18	124.67
-2300.00	92.53	173.70	130.07	132.10
-2400.00	96.90	176.99	113.75	141.73
-2500.00	100.03	-179.97	99.86	154.32
-2600.00	101.85	-177.07	90.60	170.20
-2700.00	102.31	-174.24	88.06	-171.72
-2800.00	101.40	-171.40	92.95	-154.11
-2900.00	99.15	-168.47	103.98	-139.27
-3000.00	95.61	-165.36	118.87	-127.72
-3100.00	90.87	-161.96	135.60	-118.88
-3200.00	85.05	-158.15	152.70	-112.02
-3300.00	78.32	-153.72	169.14	-106.52
-3400.00	70.91	-148.40	184.19	-101.96
-3500.00	63.14	-141.80	197.33	-98.05
-3600.00	55.44	-133.35	208.16	-94.60
-3700.00	48.45	-122.31	216.39	-91.45
-3800.00	43.06	-108.04	221.82	-88.49
-3900.00	40.29	-90.79	224.31	-85.64
-4000.00	40.86	-72.58	223.80	-82.81
-4100.00	44.63	-56.19	220.32	-79.93
-4200.00	50.67	-43.04	213.93	-76.92
-4300.00	57.99	-32.96	204.80	-73.68
-4400.00	65.77	-25.21	193.16	-70.10
-4500.00	73.46	-19.09	179.32	-66.00
-4600.00	80.67	-14.11	163.74	-61.16
-4700.00	87.11	-9.92	146.98	-55.26
-4800.00	92.59	-6.26	129.88	-47.80
-4900.00	96.94	-2.97	113.57	-38.14
-5000.00	100.06	0.07	99.73	-25.52