PHYSICS FORMULAS
2425

Types of Errors:

Personal errors due to bias or mistakes.
Systematic errors due to miscalibration of instruments, personal bias, or reaction time.
Random errors are unknown or unpredictable, such as voltage or temperature fluctuations, vibration, etc.

Accuracy - how close measurement comes to accepted value
Precision - how consistent or repeatable measurements are

Calculation of Errors:

Multiplication: operation: $A = L \times W$

error: $\Delta A = \pm(L \times \Delta W + W \times \Delta L)$

Division: operation: $D = \frac{M}{V}$

error: $\Delta D = \pm\left(\frac{\Delta M}{V} + \frac{M \times \Delta V}{V^2}\right)$

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g = 9.8 m/s^2 = 32 f/s^2
g_{moon} = 1.62 m/s^2

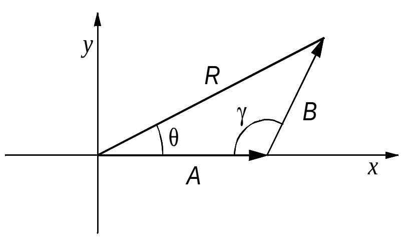
Quadratic Equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Law of Cosines:

$R = \sqrt{A^2 + B^2 - 2AB\cos\gamma}$

$\sin\theta = \frac{R}{B\sin\gamma}$



Newton's Laws:

First Law: Law of Inertia. An object at rest will remain at rest unless acted on by an external force. An object in motion will remain in motion unless . . .

Second Law: $\Sigma F = ma$, $\Sigma t = I a$ The sum of external forces on a object is equal to its mass (or inertia for rotational forces) times the acceleration.

Third Law: Every action has an equal and opposite reaction.

Law of Gravity: F = force of attraction exerted on each body

$F = G\frac{m_1 m_2}{r^2}$

G = gravitational constant 6.67 × 10^{-11} [N · m^2/kg] or [m^3/kg · s^2]

r = distance between centers [m]

Formulas for Velocity: [units: v: m/s; a: m/s^2; x: m; t: s]

$v = v_0 + at$ (for constant a)

$\bar{v} = \frac{v_0 + v}{2}$ (for constant a) average velocity

$x = v_0 t + \frac{1}{2}at^2$ (for constant a)

$v^2 = v_0^2 + 2ax$ (for constant a)

Rocket Science: The relationship between velocity and the burning of fuel.

$v_f - v_i = u \ln\frac{M_i}{M_f}$

u = speed of the exhaust relative the to rocket [m/s]

Addition of Multiple Vectors:

$\vec{R} = \vec{A} + \vec{B} + \vec{C}$ Resultant = Sum of the vectors

$\vec{R}_x = \vec{A}_x + \vec{B}_x + \vec{C}_x$ x-component $A_x = A\cos\theta$

$\vec{R}_y = \vec{A}_y + \vec{B}_y + \vec{C}_y$ y-component $A_y = A\sin\theta$

$R = \sqrt{R_x^2 + R_y^2}$ Magnitude (length) of R

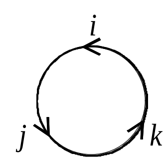
$q_R = \tan^{-1}\frac{R_y}{R_x}$ *or* $\tan q_R = \frac{R_y}{R_x}$ Angle of the resultant

Unit Vectors:

Cross Product or Vector Product:

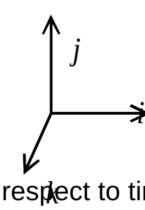
$i \times j = k$ $j \times i = -k$
$i \times i = 0$

Positive direction:



Dot Product or Scalar Product:

$i \cdot j = 0$ $i \cdot i = 1$



Velocity is the derivative of position with respect to time:

$$\mathbf{v} = \frac{d}{dt}(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) = \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j} + \frac{dz}{dt}\mathbf{k}$$

Acceleration is the derivative of velocity with respect to time:

$$\mathbf{a} = \frac{d}{dt}(v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}) = \frac{dv_x}{dt}\mathbf{i} + \frac{dv_y}{dt}\mathbf{j} + \frac{dv_z}{dt}\mathbf{k}$$

Mass/Density: [kilograms]

$M = V \times D$ mass = volume × density

Projectile Motion:

$v_{x0} = v_0 \cos q_0$ horizontal component of velocity
$v_{y0} = v_0 \sin q_0$ vertical component of velocity

$x = v_{x0} t$ horizontal distance
$v_y = v_{y0} - gt$ to find apex, let $v_y = 0$

$y = y_0 + v_{y0} t - \frac{1}{2} g t^2$ vertical distance

$y = (\tan q_0) x - \frac{g x^2}{2(v_0 \cos q_0)^2}$ vertical distance

$v_y{}^2 = v_{y0}{}^2 - 2gy$ vertical velocity

Relative Motion: $v_{PA} = v_{PB} + v_{BA}$

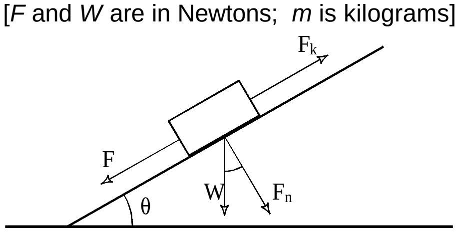
The relative velocity of object P with respect to A is equal to the velocity of P with respect to B plus the velocity of B with respect to A.

For velocities approaching the speed of light, the formula changes to: $v_{PA} = \frac{v_{PB} + v_{BA}}{1 + v_{PB} v_{BA} / c^2}$

c = the speed of light = 299,792,458 m/s

Inclined Plane: [*F* and *W* are in Newtons; *m* is kilograms]

$F = mg \sin q$
$W = mg$
$F_n = mg \cos q$
(the normal force)



$F_k = m_k F_n$ (force of **friction**, opposite the direction of movement)

$\tan q = m_k$ The coefficient of friction m_k is found when the angle q is adjusted for zero acceleration of the sliding object.

$a = g \sin q$ (acceleration)

Drag:

$D = \frac{1}{2} C r A v^2$

$v_t = \sqrt{\frac{2mg}{CrA}}$

D = Drag Force [N]
C = Coeficient of drag [ ]
r = density [kg/m^3] (air: 1.2, water: 1000)
A = effective cross-sectional area [m^2]
v = speed [m/s]
v_t = Terminal Velocity [m/s]
g = acceleration due to gravity [9.8 m/s^2]

Force: [*F* is in Newtons; *m* is kilograms]

$\text{Newtons} = \text{kg} \times \text{m} / \text{s}^2 = \frac{\text{grams} \times g}{1000} = \text{dynes} \times 10{,}000$

$\text{dynes} = \text{grams} \times \text{cm} / \text{s}^2$ 1 lb = 4.448 N

$F = ma$ force = mass X acceleration

$F = \frac{\Delta r}{\Delta t}$ $\text{force} = \frac{\text{change in momentum}}{\text{time interval}}$

$F = \frac{J}{\Delta t}$ $\text{force} = \frac{\text{impulse}}{\text{time interval}}$

conservative force - work done is independent of the path taken
non-conservative force - depends on the path taken

$F = G \frac{m_1 m_2}{r^2}$ force of gravitational attraction, where G is the constant of universal gravitation $6.673 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$

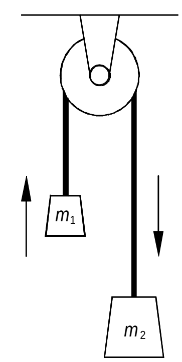
Atwood's Machine:

Acceleration in m/s^2:

$a = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) g$

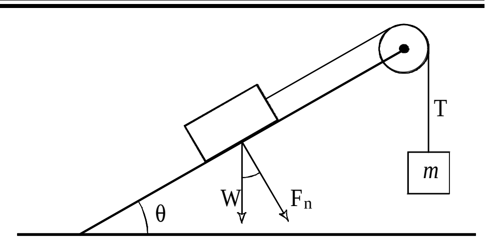
Tension in Newtons:

$T = \left(\frac{2 m_1 m_2}{m_1 + m_2} \right) g$



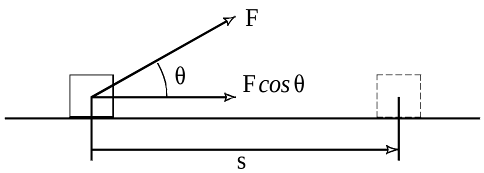
Tension:

[Newtons]



$T = m(g + a)$ (where m is accelerating upward)
$T = m(g - a)$ (where m is accelerating downward)

Work: [joules or Newton-meters]



$W = (F\cos\theta)s$ (work done on the object by F)

$W = F \bullet d$ $work = force \times displacement$

$W_g = mgy_i - mgy_f = PE_i - PE_f$ (work done by gravity, y is vertical distance in meters)

$W = KE_f - KE_i$

The work done by a conservative force on a particle is independent of the path taken.

see also: Energy, Spring

Power: [watts] $P = \frac{dW}{dt}$

Power is the rate of work. $\bar{P} = \frac{W}{\Delta t} = F\bar{v}$ $\text{watts} = \frac{\text{joules}}{\text{second}}$

Energy: [joules]

$KE = \frac{1}{2}mv^2$ (kinetic energy)

$PE = mgy$ (gravitational potential energy, y is vertical distance in meters)

$$\Delta KE = KE_f - KE_i = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 = \text{Work}$$

A falling object loses **potential** energy as it gains **kinetic** energy. In an isolated system, energy can be transferred from one type to another but total energy remains the same.

$W_{net} = \Delta KE = -\Delta PE$

$E_{total} = KE + PE$ (mechanical energy)

$PE_i + KE_i = PE_f + KE_f$ [i = initial; f = final, energy is conserved]

$mgy_i + \frac{1}{2}mv_i^2 = mgy_f + \frac{1}{2}mv_f^2$ [y = vertical distance]

$E = mc^2$ E is the mass energy, m is mass, c is the speed light 3.00×10^8 m/s

See also: Rotation and Torque

Spring: [F is in Newtons; W is in Joules; x is in meters; k is in Newtons per meter: N/m]

$F = kx$ **Hooke's Law** (force required to compress a spring with a **spring constant *k*** a distance x)

$\bar{F} = \frac{1}{2}kx$ (**average** force required to compress a spring--or **average** force output from a spring in decompression over a distance x)

$W = \frac{1}{2}kx^2$ (work done **on** a spring by an applied force)

$W = -\frac{1}{2}kx^2$ (work done **by** a spring)

$PE_s = \frac{1}{2}kx^2$ (elastic potential energy)

Simple Harmonic Motion:

$T = \frac{1}{f}$ (T is period in seconds; f is frequency in Hz)

$T = 2p\sqrt{\frac{m}{k}}$ T = period (s)
m = mass (kg)
k = spring constant (N/m)

$a = -\frac{k}{m}x$ (acceleration) x is the location in meters

$v = \pm\sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$ (A is amplitude in m; x is position)

$x = A\cos(2p\, f\, t)$ (x is position in m; f is frequency Hz)

Pendulum:

$T = 2p\sqrt{\frac{L}{g}}$ First Order Approximation for small angles
L is length in m; g is gravity

$T = 2p\sqrt{\frac{L}{g}}\left(1 + \frac{1}{4}\sin^2\frac{q}{2} + \frac{9}{64}\sin^4\frac{q}{2}\right)$ Third Order Approximation

Waves:

$v = fl$ (f is frequency in Hz; λ is wavelength in m)

$v = \sqrt{\frac{F}{m}}$ (F is tension in N; μ is mass per unit length of string in kg/m)

see also: Oscillation

Collisions:

In all collisions, momentum is conserved and the center of mass is unaffected. In an **elastic** collision, kinetic energy $KE = \frac{1}{2}mv^2$ is conserved.

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f} \quad \text{(momentum)}$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i} \quad \text{(elastic only)}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i} \quad \text{(elastic only)}$$

Momentum: [kg · m/s] $\mathbf{p} = mv$ $\sum F = \frac{d\mathbf{p}}{dt}$

Linear Momentum in a system of particles:

$\mathbf{P} = M\mathbf{v}_{cm}$ M = total mass of the system [m]
$\mathbf{v}_{cm}$ = velocity of the center of mass [m/s]

Impulse: [kg · m/s] $J = \Delta\mathbf{p} = F\Delta t = mv_f - mv_i$

impulse = force × duration *or* the change in momentum

see also **Force**

Center of Mass: The center of mass of a body or a system of bodies is the point that moves as though all of the mass were concentrated there and all external forces were applied there.

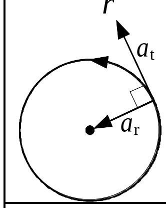
$$x_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i$$

x_{cm} = distance from origin [*m*]
M = total mass [*m*]
m = mass of object [*m*]
x = distance of object from origin [*m*]

This can be applied to *y* and *z* axis as well.

Rotation and Torque: [*q* is in radians]

$w = \frac{\Delta q}{\Delta t}$	average angular speed [rad/*s*]
$w = w_0 + at$	(if constant acceleration) [rad/*s*]
$q = w_0 t + \frac{1}{2}at^2$	(if constant acceleration) [radians]
$w^2 = w_0^2 + 2aq$	(if constant acceleration)
$\bar{a} = \frac{\Delta w}{\Delta t}$	average angular acceleration [rad/*s*^{2}]
$v_t = rw$	tangential speed [*m/s*]
$v_{cm} = rw$	velocity of the center of mass [*m/s*]

$a_t = ra$	a_t = tangential acceleration [*m/s*^{2}] r = radius [*m*] a = angular acceleration [rad/*s*^{2}]
$a_r = \frac{v_t^2}{r} = rw^2$  	a_r = radial acceleration or centripetal acceleration [*m/s*^{2}] (directed inward to center) v = speed [*m/s*] r = radius [*m*] w = angular speed [rad/*s*]
$a = \sqrt{a_t^2 + a_r^2}$	total acceleration [*m/s*^{2}]

$$F_c = ma_r = m\frac{v_t^2}{r}$$

F_c = centripetal force [N]
a_r = radial acceleration or centripetal acceleration [*m/s*^{2}] (directed inward to center)

$$T = \frac{2p\,r}{v} = \frac{2p}{w}$$

T = period [s]

Kepler's Third Law (planetary motion)

$$T^2 = \left(\frac{4p^2}{GM_s}\right)r^3 = K_s r^3$$

where T = the period
$G = 6.673\times10^{-11}\ \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$
$K_s = 2.97\times10^{-19}\ \frac{s^2}{\mathrm{m^3}}$

Torque:

$\boldsymbol{\tau} = \mathbf{r}\times\mathbf{F}$ — t = torque (vector) (positive is in the counterclockwise direction) [N · m]

$$t = rF_t = r\perp F = rF\sin f$$

t = magnitude of the torque
r = radius [*m*]
F = force [N]
$r\perp$ = perpendicular distance between axis and an extended line running through F.
f = the angle between r and F [° or rad]

$$\sum t = Ia$$

$\sum t$ = the net torque acting on a body [N · m]
I = Inertia [kg · m^2]
a = angular acceleration [rad/*s*^{2}]

Inertia: [kg · m^2] $I = \sum mr^2$ (inertia)

orbiting object:	$I = mr^2$	ring:	$I_r = \frac{1}{2}m(r_1^2 + r_2^2)$
sphere:	$I_s = \frac{2}{5}mr^2$	disk or cyl.:	$I_d = \frac{1}{2}mr^2$
thin rod (on side):	$I = \frac{1}{12}ml^2$	rod (axis end):	$I = \frac{1}{3}ml^2$

cylinder on its side (axis ctr): $I = m\left(\frac{r^2}{4} + \frac{l^2}{12}\right)$

Parallel Axis Theorem: If you know the rotational inertia of a body about any axis that passes through its center of mass, you can find its rotational intertia about any other axis parallel to that axis with the **parallel-axis theorem**:

$$I = I_{cm} + Mh^2$$

I = Inertia [kg · m^2]
I_{cm} = Inertia with axis at the center of mass [kg · m^2]
M = mass [kg]
h = distance from the center of mass to the axis [m]

Kinetic Energy: [Joules]

$KE_r = \frac{1}{2}Iw^2$ rotational kinetic energy
$KE_t = \frac{1}{2}mv^2$ translational kinetic energy
$KE = \frac{1}{2}I_{cm}w^2 + \frac{1}{2}mv_{cm}^2$ rolling kinetic energy

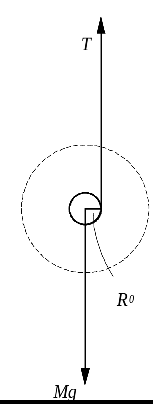
Yo-yo:

$$\sum F = T - Mg = Ma$$
$$\sum t = TR_0 = Ia$$

T = tension [N]
M = mass [kg]
R_0 = radius of axle [m]

$$a = -a\ R_0$$

$$a = \frac{-g}{1 + I/MR_0^2}$$



Angular Momentum:

$\ell = Iw$ rigid body on fixed axis [$kg \cdot m^2/s$ or $J \cdot s$]

$\ell = \mathbf{r} \times \mathbf{p} = m(\mathbf{r} \times \mathbf{v})$

ℓ = angular momentum of a particle [J · s]
r = a position vector
p = linear momentum [$kg \cdot m^2/s$ or J · s]
m = mass [kg]
v = linear velocity [m/s]

Angular momentum is conserved when torque is zero.

$I_i \omega_i = I_f \omega_f$

An Optimally Banked Curve: $\tan\theta = \frac{v^2}{rg}$

Elasticity in Length: Young's Modulus [*Pa* or N/m^2]

$$Y = \frac{F l_0}{A \Delta L}$$

F = force [*N*]
A = crossectional area [m^2]
l_0 = initial length [*m*]
ΔL = chg. in length [*m*]

Volume Elasticity: Bulk Modulus [*Pa* or N/m^2]

$$B = -\frac{V \Delta P}{\Delta V}$$

V = original vol. [m^3]
ΔV = chg. in vol. [m^3]
ΔP = change in pressure [*Pa* or N/m^2]

Pressure in a liquid: (due to gravity) [*Pa* or N/m^2]

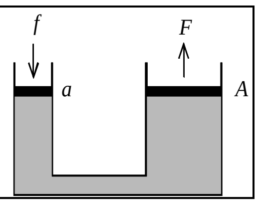
1 atm = 1.01 × 10^5 Pa = 760 torr = 14.7 in^2

$$P = P_0 + r g h$$

P_0 = atmospheric pressure if applicable [*Pa* or N/m^2]
r = density [kg/m^3]
g = gravity [m/s^2]
h = height [*m*]

$$\frac{f}{a} = \frac{F}{A}$$

f = force [*N*]
a = area [m^2]
F = force [*N*]
A = area [m^2]



Rate of Flow:

$$R = A_1 v_1 = A_2 v_2$$

R = rate of flow [m^3/s]
A = crossectional area [m^2]
v = velocity [*m/s*]

Bernoulli's Equation:

$$P_1 + \tfrac{1}{2} r v_1^2 + r g y_1 = P_2 + \tfrac{1}{2} r v_2^2 + r g y_2$$

P_1 = pressure [*Pa* or N/m^2]
v = velocity [*m/s*]
y = height [*m*]
r = density [kg/m^3]
g = gravity [m/s^2]

For a horizontal pipe: $P_1 + \tfrac{1}{2} r v_1^2 = P_2 + \tfrac{1}{2} r v_2^2$

Oscillation:

The Position Function for oscillating motion: $x = x_m \cos(w t + f)$

$$w = \frac{2p}{T} = 2p\, f$$

$$w = \sqrt{\frac{k}{m}} \text{ (spring)}$$

$$T = \frac{1}{f}$$

$$T = 2p\sqrt{\frac{m}{k}} \text{ (spring)}$$

$$T = 2p\sqrt{\frac{I}{mgh}}$$

$$F = ma = -(mw^2)x$$

x = position [m]
x_m = amplitude [m]
w = angular frequency [rad/s]
t = time [s]
f = phase angle [rad]
$(w\,t + f)$ = phase of the motion [rad]
k = spring constant [N/m]
T = period [s]
f = frequency [Hz]
F = force [N]
m = mass [kg]
I = moment of inertia [$kg \cdot m^2$]
h = distance between axis and center of mass [m]

$U(t) = \tfrac{1}{2} k x_m^2 \cos^2(w\,t + f)$ Potential Energy

$K(t) = \tfrac{1}{2} k x_m^2 \sin^2(w\,t + f)$ Kinetic Energy

$E = U + K = \tfrac{1}{2} k x_m^2$ Total Mechanical Energy

x' = velocity of the oscillating object [m/s]
x'' = acceleration of the oscillating object m/s^2]

$$\frac{d}{dx}(\cos u) = -u' \sin u \qquad \frac{d}{dx}(\sin u) = u' \cos u$$

Equations of a Line:

$y = mx + b$	slope-intercept
$Ax + By + C = 0 \quad (m = -A / B)$	first degree
$y - y_1 = m(x - x_1)$	point-slope
$Ax + By = Ax_1 + By_1$	point-slope, alt.
$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$	2-point
$\frac{x}{a} + \frac{y}{b} = 1 \quad (m = -b / a)$	intercept a = *x*-intercept b = *y*-intercept