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M 340L-C April 16, 1998

Computer Assignment 9

Eigenvalues of Hermitian Matrices

(Atlast manual, beginning on page 150)

Chapter 7, section 2, problem 8a: Eigenvalues of Symmetric Matrices

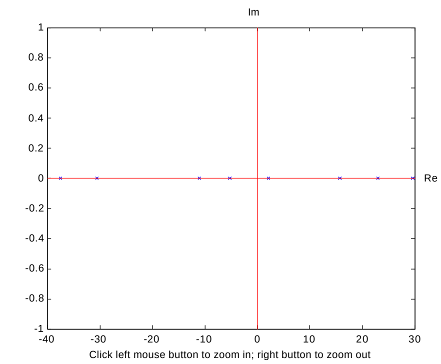
Matlab Input:

```
A = randint(2); SymmetricMatrix2x2 = A + A'
A = randint(3); SymmetricMatrix3x3 = A + A'
A = randint(4); SymmetricMatrix4x4 = A + A'
A = randint(5); SymmetricMatrix5x5 = A + A'
A = randint(6); SymmetricMatrix6x6 = A + A'
A = randint(7); SymmetricMatrix7x7 = A + A'
A = randint(8); SymmetricMatrix8x8 = A + A'
eigplot(SymmetricMatrix8x8)
```

Matlab Output:

```
SymmetricMatrix2x2 =
      12     1
       1    12
   SymmetricMatrix3x3 =
      16     3    -8
       3    16    15
      -8    15   -16
   SymmetricMatrix4x4 =
      -6     0   -15     1
       0   -12    -7    -5
     -15    -7    10    -3
       1    -5    -3    14
   SymmetricMatrix5x5 =
       0    -3     2     2    -2
      -3    -4     2     1    11
       2     2   -12   -13     5
       2     1   -13     8    -2
      -2    11     5    -2    16
   SymmetricMatrix6x6 =
      12     0     4     9    -7     3
       0     2     8     5     1    -5
       4     8     2    -1    11    -2
       9     5    -1    14    -1    12
      -7     1    11    -1   -14   -13
       3    -5    -2    12   -13    -2
   SymmetricMatrix7x7 =
     -16     4     7   -10     9     6     2
       4     4     8    -1     8    -2    -7
       7     8    -2    -3     5   -10     8
     -10    -1    -3    14    -9    10    11
       9     8     5    -9   -10    -1   -10
       6    -2   -10    10    -1    -2     2
       2    -7     8    11   -10     2     4
   SymmetricMatrix8x8 =
       4    -4    -5    -2    -8   -14   -10   -12
      -4     6    -6   -10    -5     8    -2   -11
      -5    -6   -16    -6     1    12    -2    -6
      -2   -10    -6   -14     7    -1    -2    -7
      -8    -5     1     7    14    -9     4    -2
     -14     8    12    -1    -9   -18    -9     2
     -10    -2    -2    -2     4    -9    -6     3
     -12   -11    -6    -7    -2     2     3    16
```

SymmetricMatrix8x8



Above is the eigplot of the 8x8 matrix. The eigplots of all of these matrices were viewed and in all cases the eigenvalues were real numbers. It appears that the eigenvalues of symmetric matrices having integer elements are real numbers.

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Chapter 7, section 2, problem 8b: Eigenvalues of Skew-Symmetric Matrices

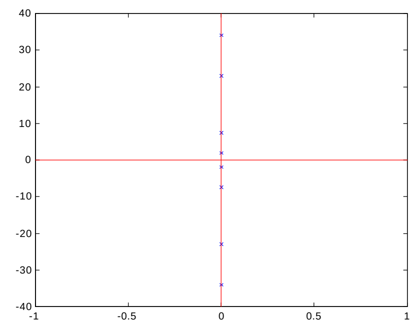
Matlab Input:

```
A = randint(2); SkewSymmetricMatrix2x2 = A - A'
A = randint(3); SkewSymmetricMatrix3x3 = A - A'
A = randint(4); SkewSymmetricMatrix4x4 = A - A'
A = randint(5); SkewSymmetricMatrix5x5 = A - A'
A = randint(6); SkewSymmetricMatrix6x6 = A - A'
A = randint(7); SkewSymmetricMatrix7x7 = A - A'
A = randint(8); SkewSymmetricMatrix8x8 = A - A'
eigplot(SkewSymmetricMatrix8x8)
```

Matlab Output:

```
SkewSymmetricMatrix2x2 =
     0   -10
    10     0
SkewSymmetricMatrix3x3 =
     0   -12    -2
    12     0     1
     2    -1     0
SkewSymmetricMatrix4x4 =
     0     4     2    12
    -4     0   -14   -14
    -2    14     0    -2
   -12    14     2     0
SkewSymmetricMatrix5x5 =
     0     5     1     5    13
    -5     0     4     2    -7
    -1    -4     0     1   -11
    -5    -2    -1     0     6
   -13     7    11    -6     0
SkewSymmetricMatrix6x6 =
     0     8     8     4    -1     0
    -8     0     4    10   -10     1
    -8    -4     0   -15    11    -1
    -4   -10    15     0     0     1
     1    10   -11     0     0     3
     0    -1     1    -1    -3     0
SkewSymmetricMatrix7x7 =
     0    -1     1   -10    -3    -6     7
     1     0    -1    -6   -11   -11    15
    -1     1     0     3    -9    -9    -7
    10     6    -3     0    -8    -5    17
     3    11     9     8     0    -8     4
     6    11     9     5     8     0    12
    -7   -15     7   -17    -4   -12     0
SkewSymmetricMatrix8x8 =
     0    -5     4    16     4    -4     5    -7
     5     0     3     0    -1     3     5   -10
    -4    -3     0     6   -14     1    -8    -4
   -16     0    -6     0     1    -4    -4     4
    -4     1    14    -1     0    17   -14    11
     4    -3    -1     4   -17     0    -7   -13
    -5    -5     8     4    14     7     0     5
     7    10     4    -4   -11    13    -5     0
```

SkewSymmetricMatrix8x8



Above is the eigplot of the 8x8 matrix. The eigplots of all of these matrices were viewed and in all cases the eigenvalues fell on the imaginary axis. It appears that the eigenvalues of skew-symmetric matrices having integer elements are either pure imaginary numbers or zero.

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Chapter 7, section 2, problem 8c: Eigenvalues of Hermitian Matrices

Matlab Input:

```
A = randint(2); SymmetricMatrix2x2 = A+A'; SkewSymMatrix2x2 = A-A';
Hermitian2x2 = SymmetricMatrix2x2 + SkewSymMatrix2x2 * i
Hermitian2x2H = conj(Hermitian2x2)'
A = randint(3); SymmetricMatrix3x3 = A+A'; SkewSymMatrix3x3 = A-A';
Hermitian3x3 = SymmetricMatrix3x3 + SkewSymMatrix3x3 * i
Hermitian3x3H = conj(Hermitian3x3)'
A = randint(4); SymmetricMatrix4x4 = A+A'; SkewSymMatrix4x4 = A-A';
Hermitian4x4 = SymmetricMatrix4x4 + SkewSymMatrix4x4 * i
Hermitian4x4H = conj(Hermitian4x4)'
A = randint(5); SymmetricMatrix5x5 = A+A'; SkewSymMatrix5x5 = A-A';
Hermitian5x5 = SymmetricMatrix5x5 + SkewSymMatrix5x5 * i
Hermitian5x5H = conj(Hermitian5x5)'
eigplot(Hermitian5x5)
```

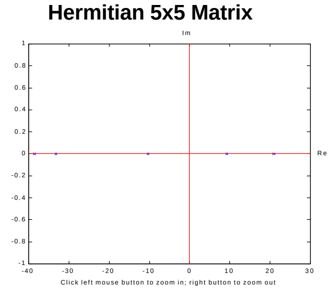
Matlab Output:

```
Hermitian2x2 =
    12   -18
   -18     0
Hermitian2x2H =
    12   -18
   -18     0
Hermitian3x3 =
  -8.0000           -17.0000 + 1.0000i  12.0000
 -17.0000 - 1.0000i  -8.0000           -12.0000 - 6.0000i
  12.0000           -12.0000 + 6.0000i        0
Hermitian3x3H =
  -8.0000           -17.0000 - 1.0000i  12.0000
 -17.0000 + 1.0000i  -8.0000           -12.0000 + 6.0000i
  12.0000           -12.0000 - 6.0000i        0
Hermitian4x4 =
   4.0000             7.0000 +11.0000i  -5.0000 + 7.0000i  -1.0000 -17.0000i
   7.0000 -11.0000i  16.0000            -7.0000 - 3.0000i  12.0000 + 6.0000i
  -5.0000 - 7.0000i  -7.0000 + 3.0000i  -8.0000            -3.0000 +15.0000i
  -1.0000 +17.0000i  12.0000 - 6.0000i  -3.0000 -15.0000i  10.0000
Hermitian4x4H =
   4.0000             7.0000 -11.0000i  -5.0000 - 7.0000i  -1.0000 +17.0000i
   7.0000 +11.0000i  16.0000            -7.0000 + 3.0000i  12.0000 - 6.0000i
  -5.0000 + 7.0000i  -7.0000 - 3.0000i  -8.0000            -3.0000 -15.0000i
  -1.0000 -17.0000i  12.0000 + 6.0000i  -3.0000 +15.0000i  10.0000
Hermitian5x5 =
  16.0000           -14.0000           -15.0000 + 1.0000i  -7.0000 - 9.0000i  -9.0000 + 9.0000i
 -14.0000           -16.0000             7.0000 - 3.0000i -13.0000 + 3.0000i        0 - 8.0000i
 -15.0000 - 1.0000i   7.0000 + 3.0000i  -2.0000                  0           -10.0000 - 4.0000i
  -7.0000 + 9.0000i -13.0000 - 3.0000i        0           -14.0000            -7.0000 + 7.0000i
  -9.0000 - 9.0000i        0 + 8.0000i -10.0000 + 4.0000i  -7.0000 - 7.0000i  -4.0000
Hermitian5x5H =
  16.0000           -14.0000           -15.0000 - 1.0000i  -7.0000 + 9.0000i  -9.0000 - 9.0000i
 -14.0000           -16.0000             7.0000 + 3.0000i -13.0000 - 3.0000i        0 + 8.0000i
 -15.0000 + 1.0000i   7.0000 - 3.0000i  -2.0000                  0           -10.0000 + 4.0000i
  -7.0000 - 9.0000i -13.0000 + 3.0000i        0           -14.0000            -7.0000 - 7.0000i
  -9.0000 + 9.0000i        0 - 8.0000i -10.0000 - 4.0000i  -7.0000 + 7.0000i  -4.0000
```

Hermitian Matrices

The eigplots of the above matrices were viewed and it was found that that the eigenvalues fell on the real axis. It appears that the eigenvalues of Hermitian matrices are real numbers.

Hermitian 5x5 Matrix



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Chapter 7, section 2, problem 12: Eigenvalues of Positive Definite Matrices

Definition: An n x n symmetric matrix A is said to be **positive definite** if $\mathbf{x}^T A\mathbf{x} > 0$ for every nonzero vector $\mathbf{x}$ in $\mathbf{R}^n$.

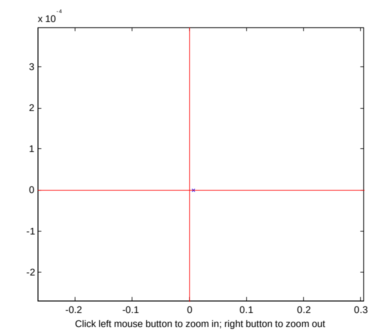
Matlab Input:

```
B = randint(4); PositiveDefinite4x4 = B' * B, format bank
Eigenvalues4x4 = eig(PositiveDefinite4x4)', format short
eigplot(PositiveDefinite4x4)
B = randint(5); PositiveDefinite5x5 = B' * B, format bank
Eigenvalues5x5 = eig(PositiveDefinite5x5)', format short
eigplot(PositiveDefinite5x5)
B = randint(6); PositiveDefinite6x6 = B' * B, format bank
Eigenvalues6x6 = eig(PositiveDefinite6x6)', format short
eigplot(PositiveDefinite6x6)
B = randint(7); PositiveDefinite7x7 = B' * B, format bank
Eigenvalues7x7 = eig(PositiveDefinite7x7)', format short
eigplot(PositiveDefinite7x7)
B = randint(8); PositiveDefinite8x8 = B' * B, format bank
Eigenvalues8x8 = eig(PositiveDefinite8x8)', format short
eigplot(PositiveDefinite8x8)
```

Matlab Output:

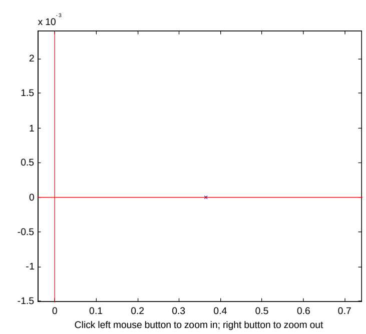
```
PositiveDefinite4x4 =
   174  -120    -8    26
  -120   102   -22   -62
    -8   -22    44    50
    26   -62    50   138
Eigenvalues4x4 =
   0.01   22.52   146.08   289.40
```

$\lambda = 0.0$




```
PositiveDefinite5x5 =
    64    11   -33    33   -15
    11   106   -59   -45     0
   -33   -59   158    31    90
    33   -45    31   195  -116
   -15     0    90  -116   168
Eigenvalues5x5 =
  57.65  74.54  0.36  241.63  316.82
```

$\lambda = 0.4$



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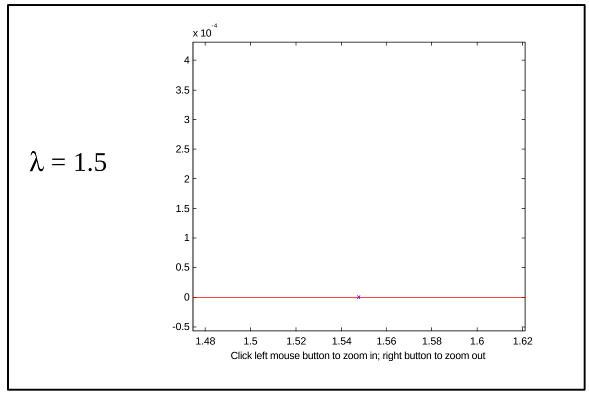
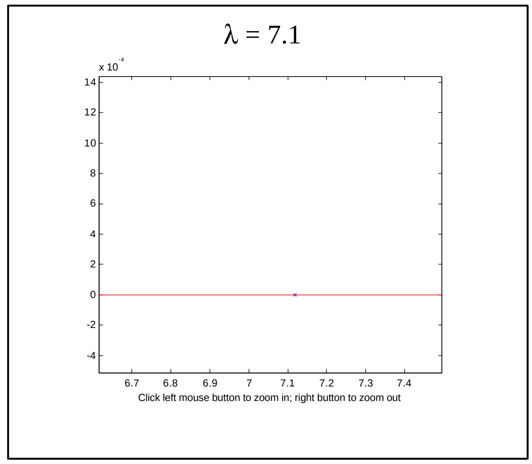
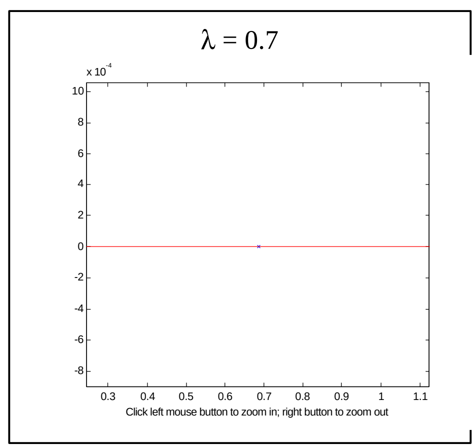



```
PositiveDefinite6x6 =
   292    35   -14   -65    12    40
    35   106    -4   -57   -38   -80
   -14    -4   321    16    73    79
   -65   -57    16    65    47    12
    12   -38    73    47   204    70
    40   -80    79    12    70   135
Eigenvalues6x6 =
  321.84  218.36  125.02  44.00  1.55  412.23
```




```
PositiveDefinite7x7 =
   303  -105  -110   -25   -58    41  -149
  -105   319   203   159   105  -166   -47
  -110   203   221   102    69  -167   -29
   -25   159   102   223    -6    52  -125
   -58   105    69    -6    97   -85    44
    41  -166  -167    52   -85   326   -28
  -149   -47   -29  -125    44   -28   207
Eigenvalues7x7 =
  21.29  7.12  39.03  88.00  313.24  772.70  454.62
```




```
PositiveDefinite8x8 =
   258     1    50    22  -107   -44   -95    37
     1   332  -243   -27    25   -63   125    71
    50  -243   375    41   -23    13    -8  -212
    22   -27    41    71   -37   -12    11    17
  -107    25   -23   -37   119    97    29   -61
   -44   -63    13   -12    97   250    54   -66
   -95   125    -8    11    29    54   348  -132
    37    71  -212    17   -61   -66  -132   225
Eigenvalues8x8 =
  9.11  0.69  53.10  132.96  199.10  321.42  556.60  705.03
```

The eigenvalues of Positive Definite matrices appear to be all positive real numbers.

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ABOUT THE MATLAB COMMANDS USED

---

apostrophe ' Matrix transpose.

Synopsis: `A'`

the linear algebraic transpose of A. For complex matrices, this involves the complex conjugate transpose.

---

eig Eigenvalues.

Synopsis:
```
e = eig(A)
[V,D] = eig(A)
```

`E = eig(A)` is a vector containing the eigenvalues of a square matrix A.

`[V,D] = eig(A)` produces a diagonal matrix *D* of eigenvalues and a full matrix *V* whose columns are the corresponding eignvectors.

---

conj Complex conjugate.

Synopsis: `Y = conj(X)`

returns the complex conjugate of the elements of X.

---

eigplot Plot eigenvalues (Atlast command)

Synopsis: `eigplot(A)`

The command `eigplot(A)` will generate a plot of the eigenvalues of the matrix A in the complex plane. To zoom in on a particular eigenvalue move the mouse so that the pointer on the screen lines up with the eigenvalue and then click the left mouse button. Keep clicking the left button until you have zoomed in to your desired accuracy. You can zoom back out by clicking on the right mouse button.

---

randint Plot eigenvalues (Atlast command)

Synopsis: `RANDINT(m,n,k,r)`

Produces an m by n matrix of rank r with integer entries in the interval [-k:k]. If less than three arguments are used the default value of k is taken to be 9. If only one input argument is used then it is assumed that the matrix is square. If the last argument is left off, no attempt is made to determine the rank.