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M 340L-C  April 2, 1998

# Computer Assignment 8

Atlast manual, beginning on page 146

## Chapter 7, section 2, problem 1a:  The Power Method

**Matlab Input:**

```
A = [0.98 0.02;0.20 0.80]
[E1, EigenvaluesOnDiag] = eig(A);
% Convert from unit eigenvectors to vectors
% with minimum element size = 1.
if E1(1,1) < E1(2,1)
  EV = [E1(1,1)/abs(E1(1,1)) E1(1,2);E1(2,1)/abs(E1(1,1)) E1(2,2)];
else
  EV = [E1(1,1)/abs(E1(2,1)) E1(1,2);E1(2,1)/abs(E1(2,1)) E1(2,2)];
end
if E1(1,2) < E1(2,2)
  EV = [EV(1,1) EV(1,2)/abs(EV(1,2));EV(2,1) EV(2,2)/abs(EV(1,2))];
else
  EV = [EV(1,1) EV(1,2)/abs(EV(2,2));EV(2,1) EV(2,2)/abs(EV(2,2))];
end
Eigenvalues = [EigenvaluesOnDiag(1,1) EigenvaluesOnDiag(2,2)]
UnitEigenvectors = E1
Eigenvectors = EV
powplot(A)
```

**Matlab Output:**

```
A =

    0.9800    0.0200
    0.2000    0.8000


Eigenvalues =

    1.0000    0.7800


UnitEigenvectors =

    0.7071   -0.0995
    0.7071    0.9950


Eigenvectors =

    1.0000   -1.0000
    1.0000   10.0000
```

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## Chapter 7, section 2, problem 1b:  The Power Method

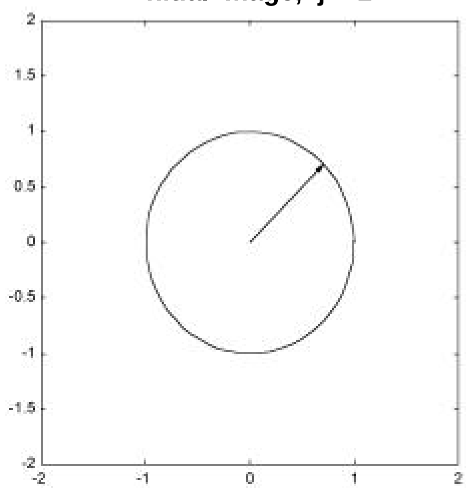
**Matlab Input:**

```
A = [0.98 0.02;0.20 0.80]
[EV EigenvaluesOnDiag] = eig(A);
Eigenvalue1 = EigenvaluesOnDiag(1,1);
Eigenvalue2 = EigenvaluesOnDiag(2,2);
Eigenvalues = [Eigenvalue1 Eigenvalue2]
RREFkernal1 = rref(A - Eigenvalue1 * eye(size(A)))
RREFkernal2 = rref(A - Eigenvalue2 * eye(size(A)))
rk1 = RREFkernal1; rk2 = RREFkernal2;
Eigenbasis = [-rk1(1,2) -rk2(1,2);rk1(1,1) rk2(1,1)]
[EV EigenvaluesOnDiag] = eig(A^25);
Eigenvalue1 = EigenvaluesOnDiag(1,1);
Eigenvalue2 = EigenvaluesOnDiag(2,2);
Eigenvaluesj25 = [Eigenvalue1 Eigenvalue2]
RREFkernal1j25 = rref(A^25 - Eigenvalue1 * eye(size(A)))
RREFkernal2j25 = rref(A^25 - Eigenvalue2 * eye(size(A)))
rk1 = RREFkernal1j25; rk2 = RREFkernal2j25;
Eigenbasisj25 = [-rk1(1,2) -rk2(1,2);rk1(1,1) rk2(1,1)]
powplot(A)
```

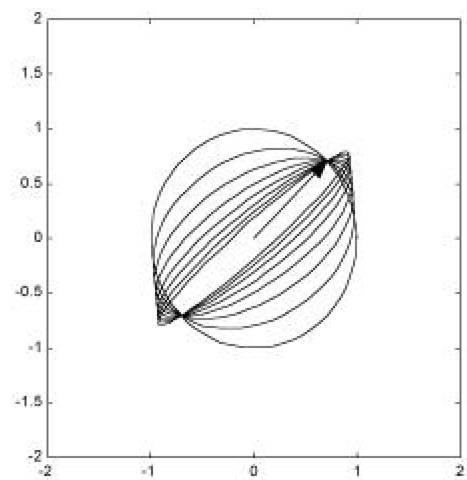
**Matlab Output:**

```
A =
    0.9800    0.0200
    0.2000    0.8000

Eigenvalues =
    1.0000    0.7800

RREFkernal1 =
    1.0000   -1.0000
         0         0

RREFkernal2 =
    1.0000    0.1000
         0         0

Eigenbasis =
    1.0000   -0.1000
    1.0000    1.0000

Eigenvaluesj25 =
    1.0000    0.0020

RREFkernal1j25 =
    1.0000   -1.0000
         0         0

RREFkernal2j25 =
    1.0000    0.1000
         0         0

Eigenbasisj25 =
    1.0000   -0.1000
    1.0000    1.0000
```

**Initial Image,  j = 1**




**Sequential Images,  j = 1,…,8**




The image of the eigenvector does not appear to change as *j* increases.  The unit circle becomes an ellip-se and its width along the major axis increases slightly.

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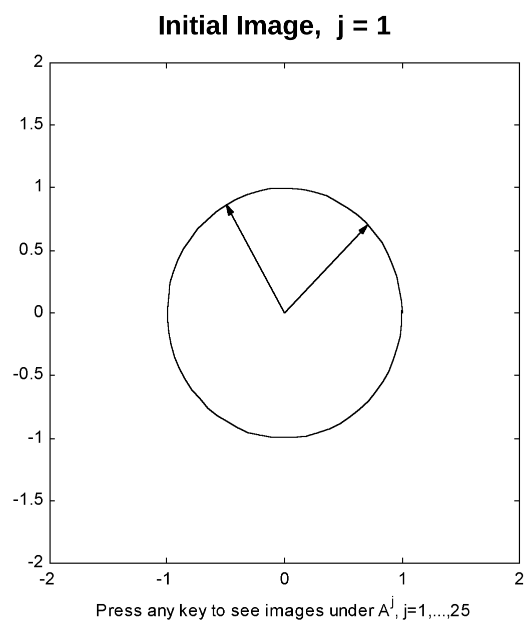
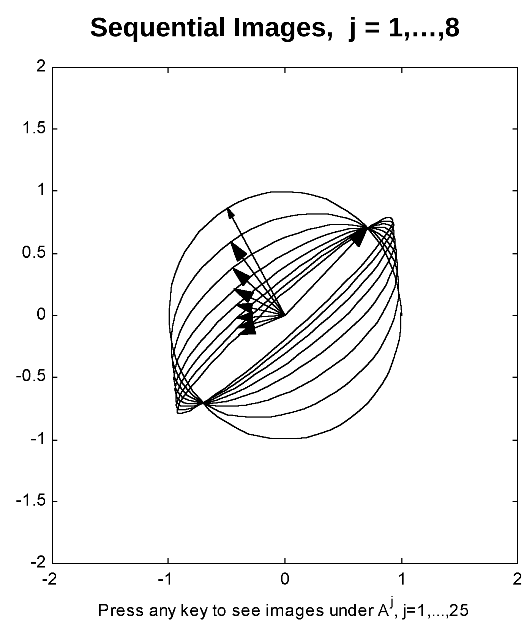
## Chapter 7, section 2, problem 1c: The Power Method

### Matlab Input:

```
A = [0.98 0.02;0.20 0.80];
x = 2*pi/3;
u = [cos(x);sin(x)];
powplot(A,u)
```

### Matlab Output:



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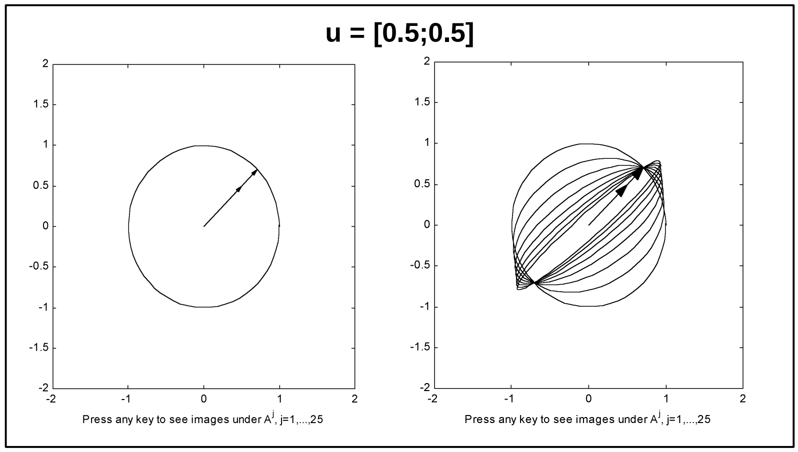
**Chapter 7, section 2, problem 1c:  The Power Method, Additional Unit Vectors**

**Matlab Input:**

```
A = [0.98 0.02;0.20 0.80];
u = [0.5;0.5];
powplot(A,u)
```

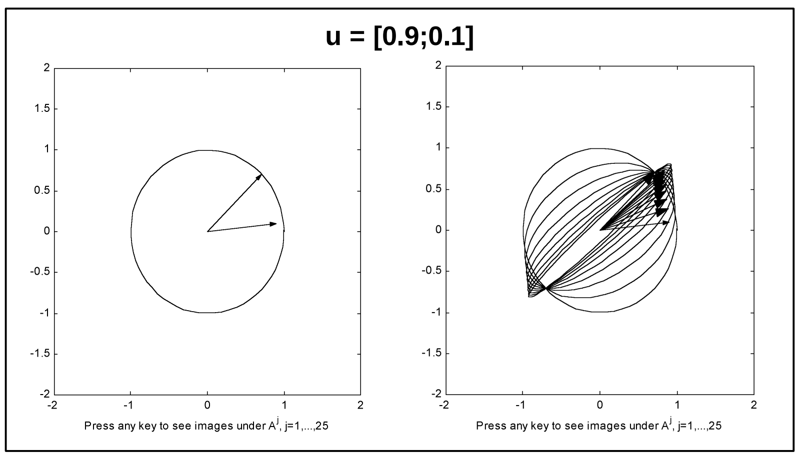
**Matlab Output:**




**Matlab Input:**

```
A = [0.98 0.02;0.20 0.80];
u = [0.9;0.1];
powplot(A,u)
```

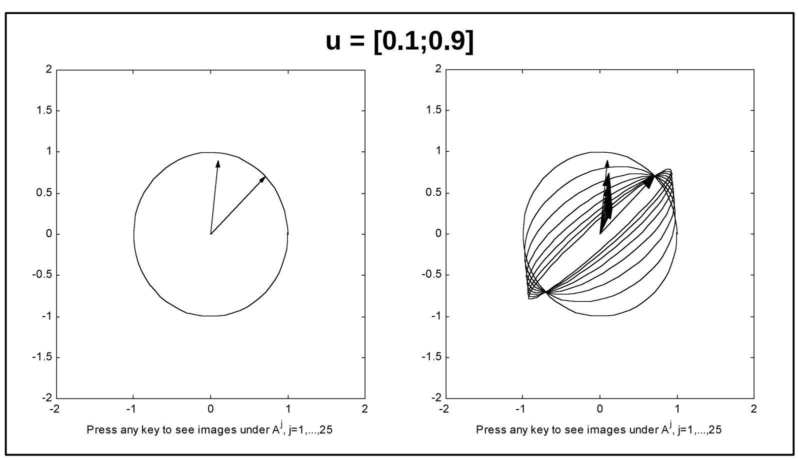
**Matlab Output:**




**Matlab Input:**

```
A = [0.98 0.02;0.20 0.80];
u = [0.1;0.9];
powplot(A,u)
```

**Matlab Output:**




The unit vector converges to the direction but not the length of the eigenvector.

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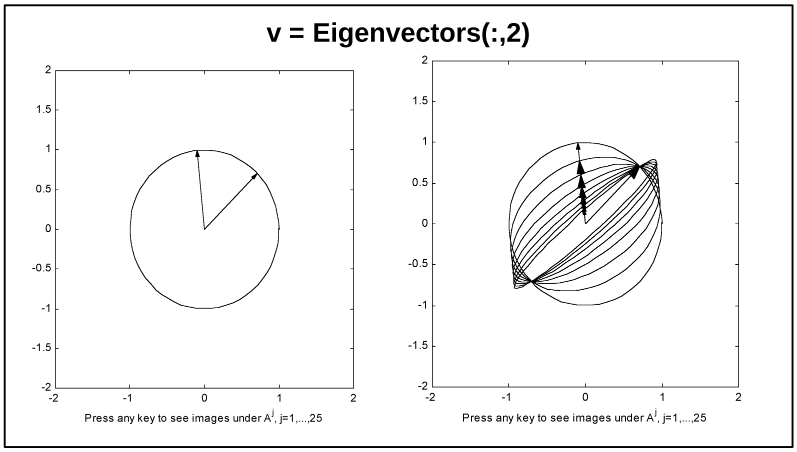
**Chapter 7, section 2, problem 1d:  The Power Method**

**Matlab Input:**

```
A = [0.98 0.02;0.20 0.80];
[Eigenvectors, EigenvaluesOnDiag] = eig(A);
Eigenvalues = [EigenvaluesOnDiag(1,1) EigenvaluesOnDiag(2,2)]
Eigenvectors
v = Eigenvectors(:,2)
ForLargeJ = A^100 * v
powplot(A,v)
```

**Matlab Output:**

```
Eigenvalues =

    1.0000    0.7800


Eigenvectors =

    0.7071   -0.0995
    0.7071    0.9950


v =

   -0.0995
    0.9950


ForLargeJ =

  1.0e-010 *

   -0.0161
    0.1612
```



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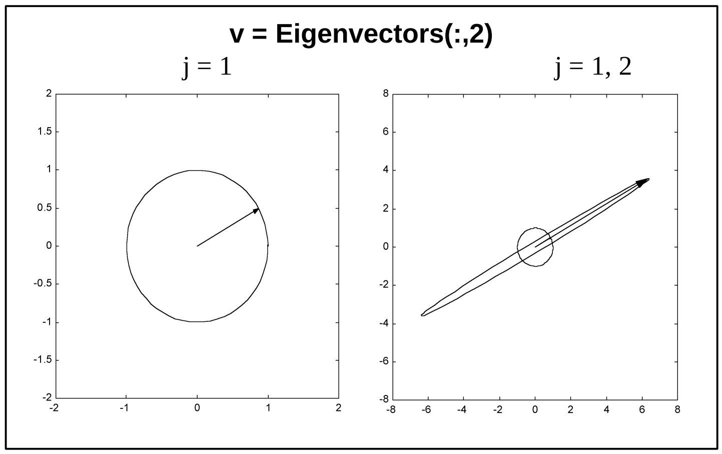
## Chapter 7, section 2, problem 1e: The Power Method, Additional Matrices

**Matlab Input:**

```
A = [5 4;3 2];
[Eigenvectors, EigenvaluesOnDiag] = eig(A);
Eigenvalues = [EigenvaluesOnDiag(1,1) EigenvaluesOnDiag(2,2)]
Eigenvectors
v = Eigenvectors(:,1)
powplot(A,v)
```

**Matlab Output:**

```
Eigenvalues =
    7.2749   -0.2749
Eigenvectors =
    0.8693   -0.6042
    0.4944    0.7968
v =
    0.8693
    0.4944
```

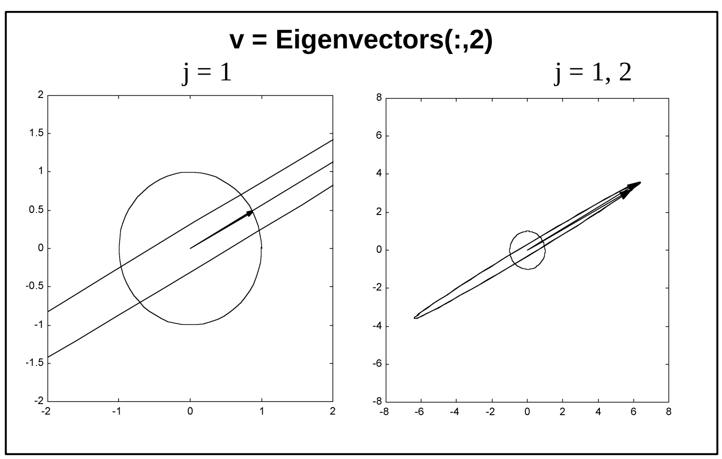



**Matlab Input:**

```
A = [4 5;2 3];
[Eigenvectors, EigenvaluesOnDiag] = eig(A);
Eigenvalues = [EigenvaluesOnDiag(1,1) EigenvaluesOnDiag(2,2)]
Eigenvectors
v = Eigenvectors(:,1)
powplot(A,v)
```

**Matlab Output:**

```
Eigenvalues =
    6.7016    0.2984
Eigenvectors =
    0.8798   -0.8037
    0.4754    0.5950
v =
    0.8798
    0.4754
```



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**Chapter 7, section 2, problem 1g:  The Power Method, The Power Method of Computing the Dominant Eigenvalue**

**Matlab Input:**

```
A = [0.98 0.02;0.20 0.80];
[Eigenvectors, EigenvaluesOnDiag] = eig(A);
Eigenvalues = [EigenvaluesOnDiag(1,1) EigenvaluesOnDiag(2,2)]
Eigenvectors
u = [1;5]
w = u;
for j = 1:40
  w = A * w;
  w = w/norm(w);
end
w
ApproxEigenvalue = w' * A * w
```

**Matlab Output:**

```
Eigenvalues =
    1.0000    0.7800
Eigenvectors =
    0.7071   -0.0995
    0.7071    0.9950
u =
     1
     5
w =
    0.7071
    0.7072
ApproxEigenvalue =
    1.0000
```

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**Chapter 7, section 2, problem 1h:  The Power Method, The Power Method of Computing the Dominant Eigenvalue, Additional Matrices and Vectors**

**Matlab Input:**

```
A = [2 3;4 5];
[Eigenvectors, EigenvaluesOnDiag] = eig(A);
Eigenvalues = [EigenvaluesOnDiag(1,1) EigenvaluesOnDiag(2,2)]
Eigenvectors
u = [1;2], w = u;
for j = 1:40
  w = A * w;
  w = w/norm(w);
end
w
```

**Matlab Output:**

```
Eigenvalues =
   -0.2749    7.2749
Eigenvectors =
   -0.7968   -0.4944
    0.6042   -0.8693
u =
     1
     2
w =
    0.4944
    0.8693
```

**Matlab Input:**

```
A = [2 1;1 5];
[Eigenvectors, EigenvaluesOnDiag] = eig(A);
Eigenvalues = [EigenvaluesOnDiag(1,1) EigenvaluesOnDiag(2,2)]
Eigenvectors
u = [2;1], w = u;
for j = 1:40
  w = A * w;
  w = w/norm(w);
end
w
```

**Matlab Output:**

```
Eigenvalues =
    1.6972    5.3028
Eigenvectors =
    0.9571    0.2898
   -0.2898    0.9571
u =
     2
     1
w =
    0.2898
    0.9571
```

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**ABOUT THE MATLAB COMMANDS USED**

---

**colon :** Create vectors, etc.

Synopsis: `v = A(:,j)`

A(:,j) is the j-th column of A

---

**eig** Eigenvalues.

Synopsis: `e = eig(A)`
`[V,D] = eig(A)`

`E = eig(A)` is a vector containing the eigenvalues of a square matrix A.

`[V,D] = eig(A)` produces a diagonal matrix *D* of eigenvalues and a full matrix *V* whose columns are the corresponding eignvectors.

---

**eye** Identity matrix.

Synopsis: `Y = eye(n)`
`Y = eye(size(A))`

`Y = eye(n)` is the n-by-n identity matrix.

`Y = eye(size(A))` is the same size as A.

---

**norm** Vector and matrix norms.

Synopsis: `v = norm(A)`

The norm of a matrix is a scalar that gives some measure of the magnitude of the elements of the matrix. Several different types of norms can be calculated. In this case, *v* is the largest singular value (eigenvalue) of A.

---

**powplot** Plot powers of a matrix (Atlast command)

Synopsis: `powplot(A,u)`

The function powplot(A) is used to demonstrate geometrically the effects of applying powers of a 2x2 matrix A to any unit vector. This is done by plotting the image of the unit circle under the transformations A^k, for k=1,...,25. If A has real eigenvalues then an eigenvector of A is also plotted. If A has a dominant eigenvalue then its eigenvector is the one that is plotted. If a unit vector u is specified as a second input argument then the images of u under the powers of A are plotted.