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Computer Assignment 4

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Chapter 4, section 1, problem 5a: Definitions of Basic Concepts

i. Put the matrix in reduced row echelon form `rref(V)`. If there are no nonzero elements other than the leading one's then the column vectors of the matrix are linearly independent.

Example:

```
V = [1 1 1;1 2 4;1 3 7;1 4 10]
R = rref(V)

V =
     1     1     1
     1     2     4
     1     3     7
     1     4    10
R =
     1     0    -2
     0     1     3
     0     0     0
     0     0     0
```

ii. $\mathbf{v}_1,\cdots,\mathbf{v}_k$ span R^n if there are at least n linearly independent vectors. In Matlab, do `rref(V)`. If the number of leading ones = n then the answer is yes. The number of leading ones gives the dimension of the image, which is also the number of linearly independent vectors required to span it.

iii. if $\mathbf{w}$ is in the span of $\mathbf{v}_1,\cdots,\mathbf{v}_k$ then there exist scalars $c_1,\cdots, c_k$ such that $c\mathbf{v}_1 + c\mathbf{v}_2 + \cdots + c\mathbf{v}_k = \mathbf{w}$. This is equivalent to saying that $V\mathbf{c} = \mathbf{w}$ where V is a matrix composed of column vectors $\mathbf{v}_1,\cdots,\mathbf{v}_k$ and $\mathbf{c}$ is a column vector containing elements $c_1,\cdots,c_k$. The Matlab command `c = solution(A,w)` will produce a result of `[]` if $\mathbf{w}$ is not in the span of $\mathbf{v}_1,\cdots,\mathbf{v}_k$ and will produce a column vector containing the required values $c_1,\cdots,c_k$ needed to express $\mathbf{w}$ as a linear combination of $\mathbf{v}_1,\cdots,\mathbf{v}_k$ if it is within the span. This kind of talk really strains my brain.

iv. Using `rref(V)` command, we can determine the rank of V by counting the leading 1's. This will be the number of linearly independent vectors required to form the basis. We then must determine which ones they are. If we are to use Matlab for that, we could test vectors individually by removing one, checking `rref` to see if there is a change in rank. If not, then that vector is not required in the basis, and testing can proceed without it.

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Chapter 4, section 1, problem 5b: Definitions of Basic Concepts

Matlab Input:

```
A = [2 3 3;-3 4 1;0 1 3;-1 2 -3], w1 = [1 0 0 1]', w2 = [1 1 0 1]'
v1 = A(:,1); v2 = A(:,2); v3 = A(:,3);
R = rref(A)
c1 = Solution(A,w1), c2 = Solution(A,w2)
Verify_w2 = .125 * v1 + .375 * v2 - .125 * v3
```

Matlab Output:

```
A =
     2     3     3
    -3     4     1
     0     1     3
    -1     2    -3
w1 =
     1
     0
     0
     1
w2 =
     1
     1
     0
     1
```

```
R =
     1     0     0
     0     1     0
     0     0     1
     0     0     0
c1 =
     []
c2 =
    0.1250
    0.3750
   -0.1250
Verify_w2 =
     1
     1
     0
     1
```

i. $\mathbf{v}_{1,}$, $\mathbf{v}_2$, $\mathbf{v}_3$ are linearly independent

ii. $\mathbf{v}_{1,}$, $\mathbf{v}_2$, $\mathbf{v}_3$ span $\Re^3$.

iii. $\mathbf{w}_1$ is not in the span of $\{\mathbf{v}_1,\cdots,\mathbf{v}_k\}$. $\mathbf{w}_2$ is in the span of $\{\mathbf{v}_1,\cdots,\mathbf{v}_k\}$.
$\mathbf{w}_2 = 0.125\mathbf{v}_1 + 0.375\mathbf{v}_2 + (-0.125)\mathbf{v}_3$

iv. The vectors $\mathbf{v}_{1,}$, $\mathbf{v}_2$, $\mathbf{v}_3$ form a basis of the span of V and the dimension is 3.

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Chapter 4, section 1, problem 5c: Definitions of Basic Concepts

Matlab Input:

```
V = [4 2 3 2 1;3 -4 -3 2 0;-1 -4 1 3 -1;3 -3 4 -2 1]
w1 = [1 0 0 1]'; w2 = [1 1 0 1]';
R = rref(V)
c1 = Solution(V,w1), c2 = Solution(V,w2)
```

Matlab Output:

```
V =
     4     2     3     2     1
     3    -4    -3     2     0
    -1    -4     1     3    -1
     3    -3     4    -2     1
R =
    1.0000         0         0         0    0.2544
         0    1.0000         0         0    0.0813
         0         0    1.0000         0    0.0430
         0         0         0    1.0000   -0.1545
c1 =
    0.1429
         0
    0.1429
         0
         0
c2 =
    0.2648
   -0.0569
    0.0128
    0.0081
         0
```

i. $\mathbf{v}_1,\cdots,\mathbf{v}_5$ are linearly dependent

ii. $\mathbf{v}_1,\cdots,\mathbf{v}_5$ span $\Re^4$.

iii. $\mathbf{w}_1$ and $\mathbf{w}_2$ are in the span of $\{\mathbf{v}_1,\cdots,\mathbf{v}_k\}$. $\mathbf{w}_1 = 0.1429\mathbf{v}_1 + 0.1429\mathbf{v}_3$
$\mathbf{w}_2 = 0.2648\mathbf{v}_1 + (-0.0569)\mathbf{v}_2 + (0.0128)\mathbf{v}_3 + (0.0081)\mathbf{v}_4$

iv. The vectors $\mathbf{v}_{1,}$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$ form a basis of the span of V and the dimension is 4.

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Chapter 4, section 1, problem 6a: Subspaces Associated with a Matrix

Matlab Input:

```
A = [1 2 5 -2 1;-1 2 3 2 3;4 -1 2 -1 2;2 -1 0 3 4]
B=rowcomb(A,1,2,1), C=rowcomb(B,1,3,-4), D=rowcomb(C,1,4,-2)
E=rowscale(D,2,1/4), F=rowcomb(E,2,1,-2)
G=rowcomb(F,2,3,9), H=rowcomb(G,2,4,5)
I=rowscale(H,3,1/7), J=rowcomb(I,3,1,2), K=rowcomb(J,3,4,-7)
```

Matlab Output:

```
A =
     1     2     5    -2     1
    -1     2     3     2     3  +I
     4    -1     2    -1     2
     2    -1     0     3     4
B =
     1     2     5    -2     1
     0     4     8     0     4
     4    -1     2    -1     2  +(-4)I
     2    -1     0     3     4
C =
     1     2     5    -2     1
     0     4     8     0     4
     0    -9   -18     7    -2
     2    -1     0     3     4  +(-2)I
D =
     1     2     5    -2     1
     0     4     8     0     4  ÷4
     0    -9   -18     7    -2
     0    -5   -10     7     2
E =
     1     2     5    -2     1  +(-2)II
     0     1     2     0     1
     0    -9   -18     7    -2
     0    -5   -10     7     2
F =
     1     0     1    -2    -1
     0     1     2     0     1
     0    -9   -18     7    -2  +9(ii)
     0    -5   -10     7     2
G =
     1     0     1    -2    -1
     0     1     2     0     1
     0     0     0     7     7
     0    -5   -10     7     2  +5(II)
H =
     1     0     1    -2    -1
     0     1     2     0     1
     0     0     0     7     7  ÷7
     0     0     0     7     7
I =
     1     0     1    -2    -1  +2(III)
     0     1     2     0     1
     0     0     0     1     1
     0     0     0     7     7
J =
     1     0     1     0     1
     0     1     2     0     1
     0     0     0     1     1
     0     0     0     7     7  +(-7)III
K =
     1     0     1     0     1
     0     1     2     0     1
     0     0     0     1     1
     0     0     0     0     0
```

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Chapter 4, section 1, problem 9: Subspaces Associated with a Matrix

Matlab Input:

```
A = randint(3,3,5,2), NulbasisA = nulbasis(A)
B = randint(3,3,5,3), NulbasisB = nulbasis(B)
C = randint(3,4,5,2), NulbasisC = nulbasis(C)
D = randint(4,3,5,3), NulbasisD = nulbasis(D)
E = randint(4,4,5,4), NulbasisE = nulbasis(E)
F = randint(4,5,5,3), NulbasisF = nulbasis(F)
G = randint(5,4,5,4), NulbasisG = nulbasis(G)
```

Matlab Output:

```
A =
     3     0     3
    -1    -3     2
     3     0     3
NulbasisA =
    -1
     1
     1
B =
     4    -2     0
     1    -3     1
     3    -4     3
NulbasisB =
   Empty matrix: 3-by-0
C =
     2     1    -1     1
    -1    -2    -1     1
    -1     1     2    -2
NulbasisC =
     1    -1
    -1     1
     1     0
     0     1
D =
     2     2    -1
     2     2     1
     1     3    -1
     1     1    -1
NulbasisD =
   Empty matrix: 3-by-0
E =
     3     1    -1    -2
    -4     3     1     2
    -5     4     2     2
     2     1    -5    -2
NulbasisE =
   Empty matrix: 4-by-0
F =
     2    -3     2    -2    -1
    -2     0     2     0    -4
     2     0    -2     0     4
    -3     5     1     1     0
NulbasisF =
    0.5000   -1.5000
         0   -1.0000
    0.5000    0.5000
    1.0000         0
         0    1.0000
G =
    -1     2     0    -2
     1    -1     2    -2
     1    -1     3    -2
    -2     2     0    -2
     2    -2     2     2
NulbasisG =
   Empty matrix: 4-by-0
```

It appears that the nulbasis is an empty matrix whenever the rank of the matrix is equal to the number of columns.

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Chapter 4, section 1, problem 10a: Coordinates with Respect to a Basis

Matlab Input:

```
U = [3 -2 3;5 0 2;-1 3 -3]
u1 = U(:,1); u2 = U(:,2); u3 = U(:,3);
w = [-3 1 5]'
Wv = solution(U,w)
VerifyW = 1*u1 + 0*u2 + (-2)*u3
```

Matlab Output:

```
U =
     3    -2     3
     5     0     2
    -1     3    -3
w =
    -3
     1
     5
Wv =
     1
     0
    -2
VerifyW =
    -3
     1
     5
```

$$w = 1\begin{bmatrix} 3 \\ 5 \\ -1 \end{bmatrix} + 0\begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} + (-2)\begin{bmatrix} 3 \\ 2 \\ -3 \end{bmatrix}$$

Chapter 4, section 1, problem 10b: Coordinates with Respect to a Basis

Matlab Input:

```
V = [-3 4 8;1 -2 1;5 -7 -11]
v1 = V(:,1); v2 = V(:,2); v3 = V(:,3);
w = [-3 1 5]'
Wu = solution(V,w)
VerifyW = 1*v1 + 0*v2 + 0*v3
```

Matlab Output:

```
V =
    -3     4     8
     1    -2     1
     5    -7   -11
w =
    -3
     1
     5
Wu =
     1
     0
     0
VerifyW =
    -3
     1
     5
```

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Chapter 4, section 1, problem 10c: Coordinates with Respect to a Basis

Matlab Input:

```
U = [3 -2 3;5 0 2;-1 3 -3], V = [-3 4 8;1 -2 1;5 -7 -11]
u1 = U(:,1); u2 = U(:,2); u3 = U(:,3);
v1 = V(:,1); v2 = V(:,2); v3 = V(:,3);
A = rot90([solution(V,u3)';solution(V,u2)';solution(V,u1)'],-1)
B = rot90([solution(U,v3)';solution(U,v2)';solution(U,v1)'],-1)
```

Matlab Output:

```
U =
     3    -2     3
     5     0     2
    -1     3    -3
V =
    -3     4     8
     1    -2     1
     5    -7   -11
A =
     7     2     3
     2     1     1
     2     0     1
B =
     1    -2    -1
     0     1    -1
    -2     4     3
```

In this problem, the matrices U and V are first entered into Matlab. In lines 2 and 3, column vectors $\mathbf{u}_1$, $\mathbf{v}_1$, etc. are formed from the columns of these matrices. In line 4, three coordinate vectors of $\mathbf{u}_3$, $\mathbf{u}_2$, and $\mathbf{u}_1$ respectively with respect to basis $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ of matrix V are determined using the `solution()` command. These are converted to row vectors and assembled into a 3×3 matrix which is rotated 90° clockwise, thus returning the rows to columns in the proper order, forming matrix A. Matrix B is formed in a similar fashion using the three coordinate vectors of $\mathbf{v}_3$, $\mathbf{v}_2$, and $\mathbf{v}_1$ respectively with respect to basis $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ of matrix U.

Chapter 4, section 1, problem 10d: Coordinates with Respect to a Basis

Matlab Input:

```
U = [3 -2 3;5 0 2;-1 3 -3]; V = [-3 4 8;1 -2 1;5 -7 -11];
u1 = U(:,1); u2 = U(:,2); u3 = U(:,3);
v1 = V(:,1); v2 = V(:,2); v3 = V(:,3);
w = [-3 1 5]'; Wu = solution(V,w), Wv = solution(U,w)
A = rot90([solution(V,u3)';solution(V,u2)';solution(V,u1)'],-1)
B = rot90([solution(U,v3)';solution(U,v2)';solution(U,v1)'],-1)
AB = A*B, BA = B*A, AWu = A*Wu, BWu = B*Wu
```

Matlab Output:

```
Wu =
     1
     0
     0
Wv =
     1
     0
    -2
A =
     7     2     3
     2     1     1
     2     0     1
B =
     1    -2    -1
     0     1    -1
    -2     4     3
AB =
     1     0     0
     0     1     0
     0     0     1
BA =
     1     0     0
     0     1     0
     0     0     1
AWu =
     7
     2
     2
BWu =
     1
     0
    -2
```