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M 340L-C February 19, 1998

Computer Assignment 3

Chapter 3, section 1, problem 1: Row Operation I

Matlab commands:

```
n=5; I=2; j=4;
A = randint(n)
B = rowswap(A,I,j)
d1 = det(A)
d2 = det(B)
```

Matlab results:

```
A =
     9     5     2    -2    -8
    -5    -1     6     8    -3
     2    -9     8     8     6
     0     6     5    -2    -9
     7    -1    -6     7    -7
B =
     9     5     2    -2    -8
     0     6     5    -2    -9
     2    -9     8     8     6
    -5    -1     6     8    -3
     7    -1    -6     7    -7
d1 =
       32693
d2 =
      -32693
```

About the Results:

In this problem a 5×5 matrix was created at random. The 2nd and 4th rows were swapped and the determinants of the two matrices were calculated. The determinants are equal in magnitude and opposite in sign.

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Row Operation I, Repeating the Problem

Matlab commands:

```
n=4; I=1; j=4;
A = randint(n)
B = rowswap(A,I,j)
d1 = det(A)
d2 = det(B)
```

Matlab results:

```
A =
    -1    -6     3     4
    -2     3    -2    -1
     7     6     6    -4
     0    -9     0    -6
B =
     0    -9     0    -6
    -2     3    -2    -1
     7     6     6    -4
    -1    -6     3     4
d1 =
        1161
d2 =
       -1161
```

Matlab commands:

```
n=6; I=3; j=4;
A = randint(n)
B = rowswap(A,I,j)
d1 = det(A)
d2 = det(B)
```

Matlab results:

```
A =
    -6    -2     6    -3    -2     6
     3     7     3     1     4     9
    -4     7     6     4     1     0
     1     2     3    -4    -1     7
    -7     0    -3     6     4    -6
     4     8    -4     1     2     9
B =
    -6    -2     6    -3    -2     6
     3     7     3     1     4     9
     1     2     3    -4    -1     7
    -4     7     6     4     1     0
    -7     0    -3     6     4    -6
     4     8    -4     1     2     9
d1 =
      -70460
d2 =
       70460
```

Matlab commands:

```
n=2; I=1; j=2;
A = randint(n)
B = rowswap(A,I,j)
d1 = det(A)
d2 = det(B)
```

Matlab results:

```
A =
    -4     7
    -5     5
B =
    -5     5
    -4     7
d1 =
    15
d2 =
   -15
```

Matlab commands:

```
n=3; I=1; j=2;
A = randint(n)
B = rowswap(A,I,j)
d1 = det(A)
d2 = det(B)
```

Matlab results:

```
A =
    -6    -4     5
    -6    -6    -1
     2    -9     8
B =
    -6    -6    -1
    -6    -4     5
     2    -9     8
d1 =
   488
d2 =
  -488
```

About the Results:

In each case, the determinants are of equal magnitude and opposite sign.

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Chapter 3, section 1, problem 2: Row Operation II

Matlab commands:

```
n=4; i=3; c=5;
A = randint(n)
B = rowscale(A,i,c)
d1 = det(A)
d2 = det(B)
```

Matlab results:

```
A =
    -7    -4    -8     0
    -9     3     9    -3
     7    -4     2    -1
    -6    -1    -1    -5
B =
    -7    -4    -8     0
    -9     3     9    -3
    35   -20    10    -5
    -6    -1    -1    -5
d1 =
        3006
d2 =
       15030
```

About the Results:

A random 4×4 matrix was created. Row 3 was multiplied by 5 using the rowscale command. As a result, the determinant is also increased by a fractor of 5.

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Row Operation II, Repeating the Problem

Matlab commands:

```
n=6; i=1; c=3;
A = randint(n)
B = rowscale(A,i,c)
d1 = det(A)
d2 = det(B)
```

Matlab results:

```
A =
     2     5     2     5    -5     5
     5     3    -9     9     3    -4
     1    -1    -2     9    -3    -1
     3     1    -4     5     9     8
    -6     6     7    -1     4     3
    -2    -8    -9     0    -2    -5
B =
     6    15     6    15   -15    15
     5     3    -9     9     3    -4
     1    -1    -2     9    -3    -1
     3     1    -4     5     9     8
    -6     6     7    -1     4     3
    -2    -8    -9     0    -2    -5
d1 =
      561696
d2 =
     1685088
```

Matlab commands:

```
n=5; i=3; c=2;
A = randint(n)
B = rowscale(A,i,c)
d1 = det(A)
d2 = det(B)
```

```
A =
     6     2    -9    -8    -9
     2    -2    -4    -9    -6
    -7     1    -9     2     2
    -6    -1    -2     2    -8
     2    -9     3    -9    -3
B =
     6     2    -9    -8    -9
     2    -2    -4    -9    -6
   -14     2   -18     4     4
    -6    -1    -2     2    -8
     2    -9     3    -9    -3
d1 =
       36154
d2 =
    72308
```

Matlab commands:

```
n=4; i=2; c=4;
A = randint(n)
B = rowscale(A,i,c)
d1 = det(A)
d2 = det(B)
```

Matlab commands:

```
A =
     2    -1     3     1
     4    -1     4    -7
     4    -3     4    -1
    -8    -7     0     4
B =
     2    -1     3     1
    16    -4    16   -28
     4    -3     4    -1
    -8    -7     0     4
d1 =
   488
d2 =
        1952
```

Matlab commands:

```
n=3; i=3; c=6;
A = randint(n)
B = rowscale(A,i,c)
d1 = det(A)
d2 = det(B)
```

```
A =
     7     7     8
    -4    -5    -5
    -5     6    -5
B =
     7     7     8
    -4    -5    -5
   -30    36   -30
d1 =
    28
d2 =
   168
```

About the Results:

In each case, the determinant is increased by the same factor used in the rowscale command.

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Chapter 3, section 1, problem 3: Row Operation III

Matlab commands:

```
n=5; i=2; j=3; c=2;
A = randint(n)
B = rowcomb(A,i,j,c)
d1 = det(A)
d2 = det(B)
```

Matlab results:

```
A =
    -9    -6    -4    -9    -6
    -8    -6    -3    -1     7
     3     9    -2     7    -5
    -6    -1     2     8     3
     7    -3    -7    -4     9
B =
    -9    -6    -4    -9    -6
    -8    -6    -3    -1     7
   -13    -3    -8     5     9
    -6    -1     2     8     3
     7    -3    -7    -4     9
d1 =
       40450
d2 =
       40450
```

About the Results:

A random 5×5 matrix was created. Two times row 2 is added to row 3. There is no change in the determinant.

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Row Operation III, Repeating the Problem

Matlab commands:

```
n=6; i=3; j=6; c=2;
A = randint(n)
B = rowcomb(A,i,j,c)
d1 = det(A)
d2 = det(B)
```

Matlab results:

```
A =
     3     7    -6    -1    -9    -6
     7     4    -1    -1     4     2
    -9     4     7    -2     3     4
    -7    -3     0     8     9    -2
     6    -6     6    -9     1    -9
    -1    -7    -1    -4    -2    -2
B =
     3     7    -6    -1    -9    -6
     7     4    -1    -1     4     2
    -9     4     7    -2     3     4
    -7    -3     0     8     9    -2
     6    -6     6    -9     1    -9
   -19     1    13    -8     4     6
d1 =
     -856479
d2 =
     -856479
```

Matlab commands:

```
n=5; i=1; j=2; c=4;
A = randint(n)
B = rowcomb(A,i,j,c)
d1 = det(A)
d2 = det(B)
```

```
A =
     5     2     3     3    -2
     6     4    -5    -2     3
     8    -6     1     2     6
     7     8     8     4    -2
    -3     1    -3    -2    -1
B =
     5     2     3     3    -2
    26    12     7    10    -5
     8    -6     1     2     6
     7     8     8     4    -2
    -3     1    -3    -2    -1
d1 =
  -353
d2 =
  -353
```

Matlab commands:

```
n=4; i=3; j=1; c=4;
A = randint(n)
B = rowcomb(A,i,j,c)
d1 = det(A)
d2 = det(B)
```

Matlab results:

```
A =
     2     5     9    -7
     1     0     6     3
     4    -6     4    -3
     0     4     0    -7
B =
    18   -19    25   -19
     1     0     6     3
     4    -6     4    -3
     0     4     0    -7
d1 =
    34
d2 =
    34
```

Matlab commands:

```
n=3; i=2; j=1; c=5;
A = randint(n)
B = rowcomb(A,i,j,c)
d1 = det(A)
d2
```

Matlab results:

```
A =
     1     9    -3
     6     9     1
     3    -8    -5
B =
    31    54     2
     6     9     1
     3    -8    -5
d1 =
   485
d2 =
   485
```

About the Results:

The rowcomb row operation has no effect on the value of the determinant.

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Chapter 3, section 1, problem 4: Scalar Multiplication

Matlab commands:

```
n=2; c=2;
A = randint(n)
d1 = det(A)
d2 = det(c*A)
```

Matlab results:

```
A =
     2     1
    -9     4
d1 =
    17
d2 =
    68
```

Matlab commands:

```
n=3; c=2;
A = randint(n)
d1 = det(A)
d2 = det(c*A)
```

Matlab results:

```
A =
     9    -1    -8
     5     3     8
     5     6     8
d1 =
  -336
d2 =
       -2688
```

Matlab commands:

```
n=4; c=2;
A = randint(n)
d1 = det(A)
d2 = det(c*A)
```

Matlab results:

```
A =
     2     4     1     3
    -5    -1    -4    -8
     7     9     7    -3
     0    -8    -3     0
d1 =
      -1096
d2 =
     -17536
```

Matlab commands:

```
n=5; c=2;
A = randint(n)
d1 = det(A)
d2 = det(c*A)
```

Matlab results:

```
A =
    -1     5    -4    -4    -3
     1     6     2    -2     0
     2     6     1     0     2
    -7     3    -8     8    -6
     8    -6    -8     2     6
d1 =
     -5202
d2 =
    -166464
```

Matlab commands:

```
n=2; c=3;
A = randint(n)
d1 = det(A)
d2 = det(c*A)
```

Matlab results:

```
A =
     9    -9
     2     6
d1 =
    72
d2 =
   648
```

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Matlab commands:

```
n=3; c=3;
A = randint(n)
d1 = det(A)
d2 = det(c*A)
```

Matlab results:

```
A =
     2    -1     6
     4    -2     6
    -8    -6    -1
d1 =
  -120
d2 =
       -3240
```

Matlab commands:

```
n=4; c=3;
A = randint(n)
d1 = det(A)
d2 = det(c*A)
```

Matlab results:

```
A =
     9    -1    -5    -2
    -7     2    -3    -1
     7     9    -6     2
     5     3     0    -8
d1 =
       -5706
d2 =
     -462186
```

Matlab commands:

```
n=5; c=3;
A = randint(n)
d1 = det(A)
d2 = det(c*A)
```

Matlab results:

```
A =
    -4     2    -3    -2    -1
     2     0    -2     0    -1
    -6    -1    -4     4    -9
     2     1    -2    -4     3
    -5     8    -4    -7     4
d1 =
    96
d2 =
       23328
```

About the Results:

Multiplying a matrix by a scalar results in an increase in the determinant by a factor equal to the scalar raised to the power of *n* where *n* is the number of rows in the square matrix. For example, if a 4×4 matrix is multiplied by a scalar 3, the determinant will be increased by a factor of 4^3.

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Chapter 3, section 1, problem 5: Transpose

Matlab commands:

```
A = randint(5)
At = transpose(A)
d1 = det(A)
d2 = det(At)
```

Matlab results:

```
A =
     8    -3    -4     1     9
     6    -4     6    -1     0
     3    -3     1     4     7
     6     1    -2     2    -6
     3     4     4     6     9
At =
     8     6     3     6     3
    -3    -4    -3     1     4
    -4     6     1    -2     4
     1    -1     4     2     6
     9     0     7    -6     9
d1 =
       35797
d2 =
       35797
```

Matlab commands:

```
A = randint(4)
At = transpose(A)
d1 = det(A)
d2 = det(At)
```

Matlab results:

```
A =
     8    -8    -7    -4
     8    -3    -6    -6
    -2     6    -6    -9
     7    -9     2     5
At =
     8     8    -2     7
    -8    -3     6    -9
    -7    -6    -6     2
    -4    -6    -9     5
d1 =
   231
d2 =
   231
```

Matlab commands:

```
A = randint(3)
At = transpose(A)
d1 = det(A)
d2 = det(At)
```

Matlab results:

```
A =
     1    -4    -5
    -5     5     2
     6     3     2
At =
     1    -5     6
    -4     5     3
    -5     2     2
d1 =
   141
d2 =
   141
```

Matlab commands:

```
A = randint(2)
At = transpose(A)
d1 = det(A)
d2 = det(At)
```

Matlab results:

```
A =
    -1    -1
     8    -2
At =
    -1     8
    -1    -2
d1 =
    10
d2 =
    10
```

About the Results:

The determinants of A and A^T are equal.

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Chapter 3, section 1, problem 6: Matrix Addition and Multiplication

Matlab commands:

```
A = randint(5), B = randint(5)
d1 = det(A + B)
d2 = det(A) + det(B)
d3 = det(A*B)
d4 = det(A) * det(B)
```

Matlab results:

```
A =
     4    -3     0     7     6
     4    -7     1    -4     8
    -8     3    -7    -5    -5
    -1     4    -1     7    -5
    -1     4     4    -5    -9
B =
    -8    -6    -3    -1     7
     3     9    -2     7    -5
    -6    -1     2     8     3
     7    -3    -7    -4     9
    -6    -4    -9    -6     3
d1 =    36374
d2 =    15962
d3 = -2954535
d4 = -2954535
```

Matlab commands:

```
A = randint(4), B = randint(4)
d1 = det(A + B)
d2 = det(A) + det(B)
d3 = det(A*B)
d4 = det(A) * det(B)
```

Matlab results:

```
A =
     5     3    -7    -2
    -4    -5    -6     1
    -1     6     2    -1
     8     2     2    -9
B =
    -9     3     2     2
    -4    -8    -9    -8
    -9    -9    -9    -3
    -2     2    -6     2
d1 =     -8216
d2 =      1375
d3 = -12462156
d4 = -12462156
```

Matlab commands:

```
A = randint(3)
B = randint(3)
d1 = det(A + B)
d2 = det(A) + det(B)
d3 = det(A*B)
d4 = det(A) * det(B)
```

Matlab results:

```
A =
     7     6     4
    -9    -1     4
    -7     7    -3
B =
    -6    -1     6
    -7     7    -1
    -6     0    -1
d1 =    -900
d2 =    -490
d3 = -231575
d4 = -231575
```

Matlab commands:

```
A = randint(2)
B = randint(2)
d1 = det(A + B)
d2 = det(A) + det(B)
d3 = det(A*B)
d4 = det(A) * det(B)
```

Matlab results:

```
A =
    -1     8
    -2    -9
B =
    -4     4
    -9     3
d1 =   162
d2 =    49
d3 =   600
d4 =   600
```

About the Results:

The results suggest that
det(A+B) ≠ det(A) + det(B) and that
det(A×B) = det(A) × det(B)

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Chapter 3, section 1, problem 7: Inverses

Matlab commands:

```
A = randint(5), Ai = inv(A)
d1 = det(A)
d2 = det(Ai)
```

Matlab results:

```
A =
     9     4     6     4    -5
     1    -2     8    -6     1
    -2    -9     7     8     8
    -6    -2    -3     1    -3
     2     5     2     3     3
Ai =
    0.0289   -0.0608    0.0029   -0.1714   -0.1107
   -0.0068    0.0273   -0.0445    0.0631    0.1614
    0.0364    0.0988    0.0161    0.0910    0.0758
    0.0431   -0.0566    0.0403    0.0541    0.0372
   -0.0753   -0.0143    0.0212   -0.1056    0.0505
d1 =
       52908
d2 =
  1.8901e-005
```

Matlab commands:

```
A = randint(4), Ai = inv(A)
d1 = det(A)
d2 = det(Ai)
```

Matlab results:

```
A =
    -2    -2    -1     0
     2     3     2     5
     4     6     1     0
    -2    -2     4    -6
Ai =
   -1.3529   -0.1765   -0.4118   -0.1471
    0.9265    0.0882    0.4559    0.0735
   -0.1471    0.1765   -0.0882    0.1471
    0.0441    0.1471   -0.0735   -0.0441
d1 =
  -136
d2 =
   -0.0074
```

Matlab commands:

```
A = randint(3)
Ai = inv(A)
d1 = det(A)
d2 = det(Ai)
```

Matlab results:

```
A =
     4     4     3
     9     0    -3
     6    -7    -7
Ai =
    0.2258   -0.0753    0.1290
   -0.4839    0.4946   -0.4194
    0.6774   -0.5591    0.3871
d1 =
   -93
d2 =
   -0.0108
```

Matlab commands:

```
A = randint(2)
Ai = inv(A)
d1 = det(A)
d2 = det(Ai)
```

Matlab results:

```
A =
     1     3
     6     9
Ai =
   -1.0000    0.3333
    0.6667   -0.1111
d1 =
    -9
d2 =
   -0.1111
```

About the Results:

The results suggest that the determinant of the inverse matrix equals the inverse of the determinant of the original matrix.
$\mathbf{det(A^{-1}) = det(A)^{-1}}$
This property is reminiscent of $\mathbf{det(A \times B) = det(A) \times det(B)}$ found in Exercise 6.

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Chapter 5, section 2, problem 1a: Finding Bases for the Image and Kernel

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Chapter 5, section 2, problem 1b: Finding Bases for the Image and Kernel

```
A = [1 2 0 1 -1;2 1 3 1 0;-1 0 -2 0 1;0 0 0 2 8]
C = [A; eye(5)]
D = rref(C')'
```

```
A =
     1     2     0     1    -1
     2     1     3     1     0
    -1     0    -2     0     1
     0     0     0     2     8
C =
     1     2     0     1    -1
     2     1     3     1     0
    -1     0    -2     0     1
     0     0     0     2     8
     1     0     0     0     0
     0     1     0     0     0
     0     0     1     0     0
     0     0     0     1     0
     0     0     0     0     1
D =
   1.0000         0         0         0         0
        0    1.0000         0         0         0
        0         0    1.0000         0         0
  -2.0000    4.0000    6.0000         0         0
        0         0         0    1.0000         0
        0         0         0         0    1.0000
  -0.2000    0.2000   -0.2000   -0.4000    0.2000
   0.6000    0.4000    0.6000   -0.8000   -1.6000
  -0.4000    0.4000    0.6000    0.2000    0.4000
```