LINEAR SYSTEMS DEFINITIONS

A Glossary of Terms

abscissa of convergence – see *region of convergence*

additivity property - a sum of causes results in a sum of their respective effects. see linearity.pdf

aliasing - a type of distortion found in signal sampling which may be viewed in a graph of the signal in the time domain as overlapping tails. It may be eliminated by bandlimiting the signal before sampling.

amplitude spectrum - the plkiot of the amplitude versus the (radian) frequency ω. A point is plotted for each harmonic ($n = 0, 1, 2, 3, \cdots$)

asymptotically stable - describes a system which, in the absence of input, will return to a zero-state over time given non-zero initial conditions; from a mathematical view this occurs when the real parts of all characteristic roots are negative

bandwidth - the difference between the highest and lowest frequencies in the signal.

BIBO - bounded-input bounded-output. a definition of stability which says that a system is stable iff a bounded input produces a bounded output

causal - describes a system whose output at the present instant depends only upon the present and past values of the input

characteristic equation - the characteristic polynomial set equal to zero

characteristic mode - a mathematical term in the form $e^{\lambda t}$ where λ is a characteristic root and e is the natural number. The sum of the characteristic modes comprises the natural response of a system. In the case of repeated roots, characteristic modes will be of the form $e^{\lambda t}$, $te^{\lambda t}$, $t^2e^{\lambda t}$, $t^3e^{\lambda t}$, etc.

characteristic polynomial - A linear system may be described by an equation in the form $Q(D)y(t) = P(D)f(t)$ where $f(t)$ is the input, $y(t)$ is the output. $Q(D)$ and $P(D)$ are polynomials of the D (differential) operator. When λ is substituted for the D operator in $Q(D)$, the result $Q(\lambda)$ is then the characteristic polynomial.

characteristic roots - the value(s) assumed by λ when the characteristic equation is solved.

continuous-time - describes a function whose values are defined continuously over a period of time

convolution integral - the zero-state response $y(t)$ expressed as an integral $f_1(t) * f_2(t) \equiv \int_{-\infty}^{\infty} f_1(\tau) f_2(t-\tau)d\tau$

decomposition property - permits the separation of an output into components caused by initial conditions and input

discrete-time - describes a function whose values are defined only at discrete instants of time (as opposed to continuous)

distributed-parameter - describes a system in which the physical dimensions cannot be assumed to be zero, so that signals are considered a function of space as well as time. This leads to mathematical models consisting of partial differential equations.

dynamic - describes a system whose output depends on more than the value(s) of it's inputs at a given instant

forced response - mathematically those terms of the total response which are not characteristic modes, comprised of the non-characteristic mode zero-state response terms. The forced response is the reaction of the circuit due to excitation.

frequency-domain analysis - a method for solving time-domain equations, i.e. $f(t) = \ldots$, by converting to the frequency or s-domain by taking the Laplace transform of the equation, then reducing the equation, followed by a conversion back to the time-domain. The parameter s (generally a complex number) is the complex frequency of the signal e^{st}.

fundamental frequency - the largest positive number of which all the represented frequencies are integral multiples. This number is equal to the **greatest common factor** (GCF) in their numerators divided by the **least common multiple** (LCM) of their denominators. i.e. for the set $\{\frac{2}{3}, \frac{6}{7}, 2\}$, the GCF of the numerators (2, 6, 2) is 2 and the LCM of the denominators (3, 7, 1) is 21. Therefore $\frac{2}{21}$ is the largest number of which $\frac{2}{3}$, $\frac{6}{7}$, and 2 are integral multiples.

harmonically related - Two frequencies are **harmonically related** when their ratio is a rational number, i.e. doesn't include π or a square root, etc.

homogeneity property - if a cause is increased k-fold, the effect also increases k-fold. see linearity.pdf

impulse response – The impulse response $h(t)$ is the system response to an impulse input $\delta(t)$ applied at $t = 0$ with all the initial conditions zero at $t = 0$.

instantaneous - describes a system whose output at any time t depends at most on the value(s) of its inputs at the same instant. Opposite of **dynamic**.

Laplace transform - a formula by which a function in the time-domain may be converted into a function in the frequency- or s-domain. $F(s) = \int_{-\infty}^{\infty} f(t)e^{-st}dt$

linear system - a system whose output is proportional to its input and exhibits the superposition property. see linearity.pdf

lumped-parameter - describes a system in which each component is regarded as being lumped at one point in space. For example in an electrical system, since current travels so quickly, it can be assumed that the value of the current is equal in all parts of a component.

LTI - linear time-invariant (system)

LTIC - linear time-invariant continuous-time (system)

LTID - linear time-invariant discrete-time (system)

marginally stable - describes a system which, in the absence of input, produces a steady non-zero output or a bounded oscillatory output over time in response to non-zero input conditions; mathematically this occurs when there are non-repeated roots with a 0 real component and there are no roots with a positive real component

MIMO - multiple-input, multiple-output (system)

natural response - mathematically those terms of the total response which are characteristic modes, comprised of all of the zero-input response terms and some of the zero-state response terms. The natural response is the reaction of the circuit without excitation.

Nyquist interval - the minimum sampling interval required to recover the original signal from its samples, equal to the reciprocal of twice the bandwidth

Nyquist rate - the minimum sampling rate required to recover the original signal from its samples, equal to 2× the bandwidth

periodic function - a function in which all frequencies are **harmonically related**. This means that the ratio of any two frequencies must be a rational number.

phase spectrum - the plot of the phase θ_n versus the (radian) frequency ω.

region of convergence – the area of the complex plane in which the Laplace transform of a particular function does not go to positive or negative infinity. This is called a **right half-plane** and consists of the area to the right of a vertical line called the **abscissa of convergence**.

repeated roots - equal characteristic roots which occur when the characteristic equation contains a term involving the *D* (differential) operator that is raised to a power. For example the term $(D + 2)^3$ would result in 3 characteristic roots λ having the same value of -2.

SISO - single-input, single-output (system)

superposition - a combination of the properties of additivity and homogeneity. see linearity.pdf

system - an entity that processes a set of signals (inputs) to yield another set of signals (outputs or response)

time-domain analysis - a method of solving system equations directly using *t* as the independent variable

time-invariant - describes a system whose output will experience no change other than an equal shift in time when the input is shifted in time

time-shift – the expression $f(t - t_0)$ is a time-shift wherein the function $f(t)$ is shifted graphically to the **right** by t_0, in effect delaying the signal by t_0.

total response - sum of the zero-input response and the zero-state response

unit impulse function - an input function whose graph depicts a single positive excursion with relatively small width and large amplitude with an area of 1.

Defined: $\delta(t) = 0,\ t \neq 0;\ \int_{-\infty}^{\infty} \delta(t)\, dt = 1$

When a function is multiplied by an impulse, the area under the product of the function with an impulse is equal to the value of that function at the instant where the unit impulse is located. If we know the response of a system to a unit impulse, we can determine the system response to an arbitrary input $f(t)$.

zero-input response - the output component resulting only from the system's initial conditions, with the input held to zero; mathematically composed exclusively of characteristic mode terms

zero-state response - the output component resulting only from the system's input, with the initial conditions assumed to be zero; mathematically composed of both characteristic and non-characteristic mode terms