# MODULATION

This document covers the basics of amplitude modulation AM and frequency modulation FM:

## AMPLITUDE MODULATION, AM

For conventional AM:

- Though the carrier component contains most of the transmitted power, it contains no signal information.
- The sideband information is redundant. Both sidebands contain amplitude and frequency information about the message.
- For a given bandwidth of channel (10KHz by law) the highest legal message signal frequency is 5KHz.
- The envelope is the message, so an envelope detector is a sufficient demodulator.

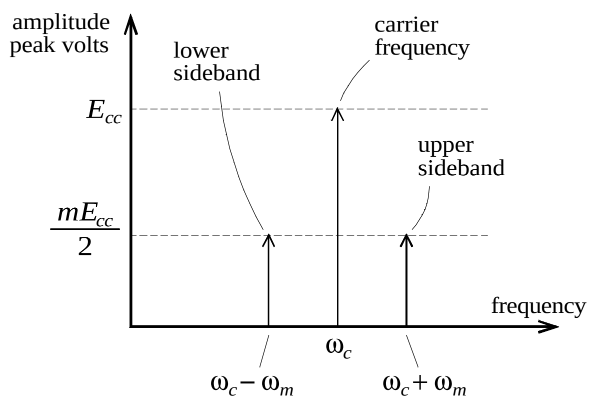
**Expressions for the amplitude modulated signal:**

$$v_{out} = E_{cc}\left[1 + m\cos\omega_m t\right]\cos\omega_c t$$

$$v_{out} = \underbrace{E_{cc}\cos\omega_c t}_{\text{carrier}} + \underbrace{\frac{mE_{cc}}{2}\cos\left(\omega_c + \omega_m\right)t}_{\text{upper sideband}} + \underbrace{\frac{mE_{cc}}{2}\cos\left(\omega_c - \omega_m\right)t}_{\text{lower sideband}}$$

Amplitudes are in peak volts.

**Carrier frequency and sidebands:**




| **Modulation index:** The ratio of the modulated signal amplitude to the carrier signal amplitude. | $m \equiv \frac{E_m}{E_{cc}}$ |
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| **Lower sideband frequency:** | $f_{LSB} = f_c - f_m = \frac{\omega_c}{2\pi} - \frac{\omega_m}{2\pi}$ |
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| **Minimum bandwidth required:** | $BW_{\min} = f_{\text{USB}} - f_{\text{LSB}} = \dfrac{2\omega_m}{2\pi}$ |

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| **Carrier power:** | $P_c = \dfrac{\left(v_{\text{carrier (RMS)}}\right)^2}{R_L}, \quad \text{where } v_{\text{RMS}} = \dfrac{v_{\text{peak}}}{\sqrt{2}}$ |

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| **One sideband power:** | $P_{\text{SB}} = \dfrac{\left(\frac{1}{2} m v_{\text{carrier (RMS)}}\right)^2}{R_L}, \quad \text{where } v_{\text{RMS}} = \dfrac{v_{\text{peak}}}{\sqrt{2}}$ |

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| **Total power to load:** | $P_{\text{total}} = P_c\left(1 + \dfrac{m^2}{2}\right)$ |

## FREQUENCY MODULATION, FM

For FM:

- The maximum frequency deviation (FCC regulation) is 75 KHz.
- Station frequency spacing is 200 KHz.

**Expressions for the frequency modulated signal:**

$$v_{out} = E_{cc}\left[\omega_c t + M_f \sin \omega_m t\right]$$

$$v_{out} = E_{cc}\begin{Bmatrix} J_0\left(M_f\right)\cos\omega_c t + J_1\left(M_f\right)\left[\cos\left(\omega_c + \omega_m\right)t - \cos\left(\omega_c - \omega_m\right)t\right] \\ +J_2\left(M_f\right)\left[\cos\left(\omega_c + 2\omega_m\right)t + \cos\left(\omega_c - 2\omega_m\right)t\right] \\ +J_3\left(M_f\right)\left[\cos\left(\omega_c + 3\omega_m\right)t - \cos\left(\omega_c - 3\omega_m\right)t\right] + \cdots \end{Bmatrix}$$

Even for sinusoidal modulation the number of side frequency pairs is theoretically infinite. The $J_n\left(M_f\right)$ functions are Bessel functions. Amplitudes are in peak volts.

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| **The instantaneous frequency:**<br><br>$f_i = f_c + \Delta f \cos\omega_m t \quad \text{Hz}$ | $f_i$ is the instantaneous frequency [Hz]<br>$f_c$ is the carrier frequency [Hz]<br>$\Delta f$ is the frequency deviation [Hz]<br>$\omega_m$ is the modulating signal frequency or message signal [rad/s] |

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| **Modulation index:**<br><br>The ratio of the frequency deviation to the modulating signal frequency. | $M_f \equiv \dfrac{\Delta f}{f_m}$ |

| **Minimum bandwidth required:** $BW_{\min} = 2nf_m$ | where $n \equiv M_f + 1$ rounded to the next integer |
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