

CREATING BODE PLOTS FROM A TRANSFER FUNCTION

Given the transfer function:

$$G(s) = \frac{10s^2 \left(1 + \frac{s}{10^2}\right) \left(1 + \frac{s}{10^6}\right)^2}{\left(1 + \frac{s}{10}\right)^2 \left(1 + \frac{s}{10^3}\right) \left(1 + \frac{s}{10^5}\right)^4}$$

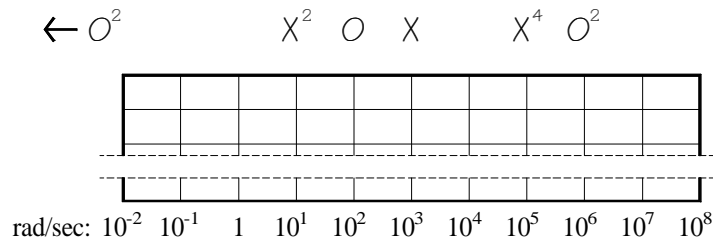
where $s = j\omega$
and G is the gain of the system in V/V.

Determine the zeros and poles of the function:

We find the **zeros** and **poles** by observing the numerator and denominator respectively, and determining the value of s that will make each term zero. The absolute value of that term is the zero or pole. The power to which the expression is raised corresponds to the **order** of the pole or zero.

Zeros (from numerator)	Poles (from denominator)
$s = 0 \rightarrow 2^{\text{nd}}$ order zero at $\omega = 0$	$s = -10 \rightarrow 2^{\text{nd}}$ order pole at $\omega = 10$
$s = -10^2 \rightarrow 1^{\text{st}}$ order zero at $\omega = 10^2$	$s = -10^3 \rightarrow 1^{\text{st}}$ order pole at $\omega = 10^3$
$s = -10^6 \rightarrow 2^{\text{nd}}$ order zero at $\omega = 10^6$	$s = -10^5 \rightarrow 4^{\text{th}}$ order pole at $\omega = 10^5$

Mark the locations of the zeros and poles using Os and Xs respectively at the top of the Bode Frequency plot.

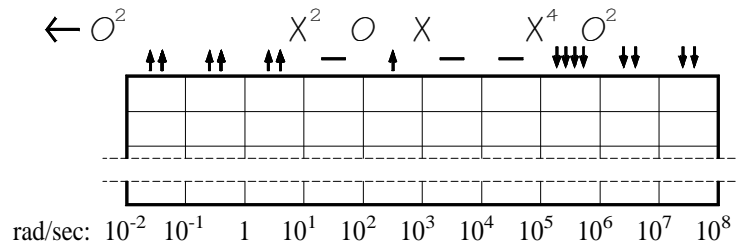


Note that the location of the first zero is off the graph to the left since the $\omega = 0$ location is not available on the logarithmic scale. The Os and Xs mark the frequencies at which the slope of the graph will change, the **frequency corners**.

Determining the slope of the Bode Amplitude Plot:

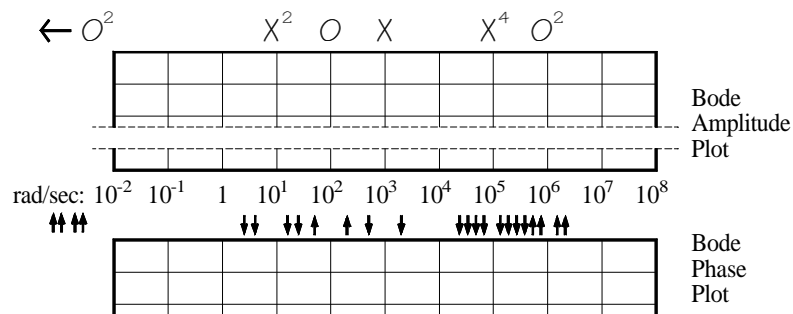
The order value of each zero and pole indicates the change in slope in multiples of 20 dB/decade. For example, an order of 2 means there will be change in slope of 40 dB/decade at the frequency of the zero or pole. The slope is increased at zeros and reduced at poles.

Beginning at the left of the graph, mark the top of each decade column with up or down arrows indicating the slope at that decade. Each arrow stands for 20 dB/decade of slope. In this case, since there is a 2nd order zero at $\omega = 0$, which is off the graph to the left, we begin by marking the first decade with two up arrows.



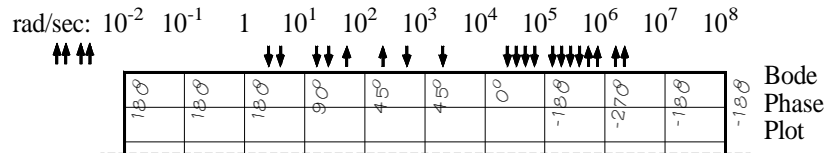
Determining the slope of the Bode Phase Plot:

On the Bode Phase Plot, again use up and down arrows to mark the slope of the graph. This time, each arrow represents a 45° /decade slope for each order of zero or pole. Zeros cause an upward slope and poles cause a downward slope. Each pole or zero exerts an influence over one decade on either side of the pole or zero frequency. For example, a 2nd order pole causes a 90° /decade downward slope that extends from one decade below to one decade above the pole frequency. This would be indicated on the graph by two downward arrows on each side of the pole frequency. Some decades will have both up and down arrows. These will be summed, with the result that some arrows will cancel each other.



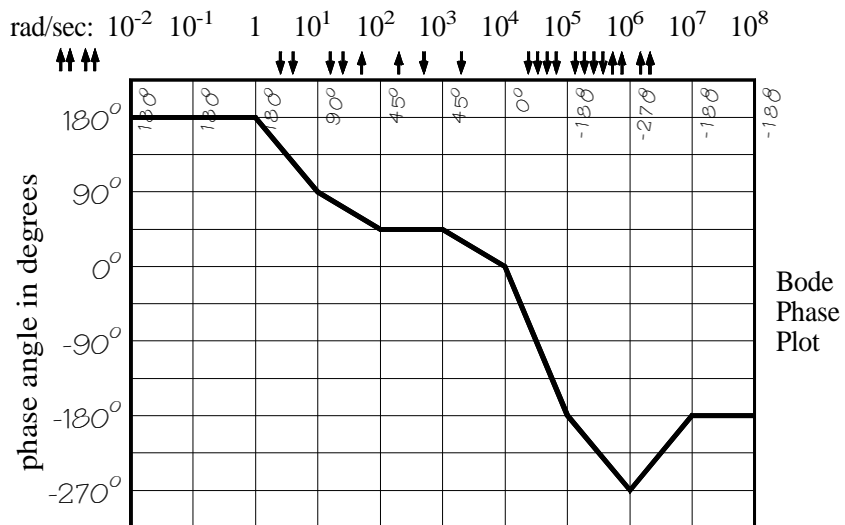
Determining the phase angles for the Bode Phase Plot:

Next, we mark the graph with the phase angle at each decade. The four up arrows due to the 2nd order zero at $\omega = 0$ tell us that the plot begins at $+180^\circ$. The phase angle remains unchanged until we encounter the next two down arrows which cause a -90° shift to $+90^\circ$. The following decade has two up arrows and one down arrow, resulting in a net -45° phase change, etc.



Drawing the Bode Phase Plot:

We now have all the information we need to label the vertical axis and draw the Bode Phase Plot.



Determining the starting point for the Bode Amplitude Plot:

We have the information we need to draw the Bode Amplitude Plot except for the amplitude value of the starting point. Since our plot begins at $\omega = 10^{-2}$, we need the amplitude for $G(j10^{-2})$. Since we are only concerned with the amplitude and not the phase, the j can be ignored. Plugging in to the transfer function we have:

$$G(10^{-2}) = \frac{10(10^{-2})^2 \left(1 + \frac{(10^{-2})}{10^2}\right) \left(1 + \frac{(10^{-2})}{10^6}\right)^2}{\left(1 + \frac{(10^{-2})}{10}\right)^2 \left(1 + \frac{(10^{-2})}{10^3}\right) \left(1 + \frac{(10^{-2})}{10^5}\right)^4}$$

Notice that there is only one term that is not approximately equal to 1. So we have:

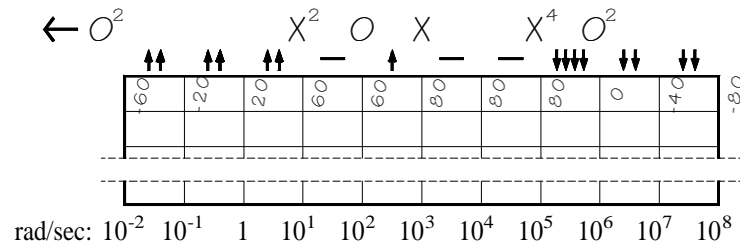
$$G(10^{-2}) \approx 10(10^{-2})^2 = 10^{-3} \text{ V/V}$$

Now we convert to units of dB:

$$G_{dB} = 20 \log(G_{V/V}) = 20 \log(10^{-3}) = -60 \text{ dB}$$

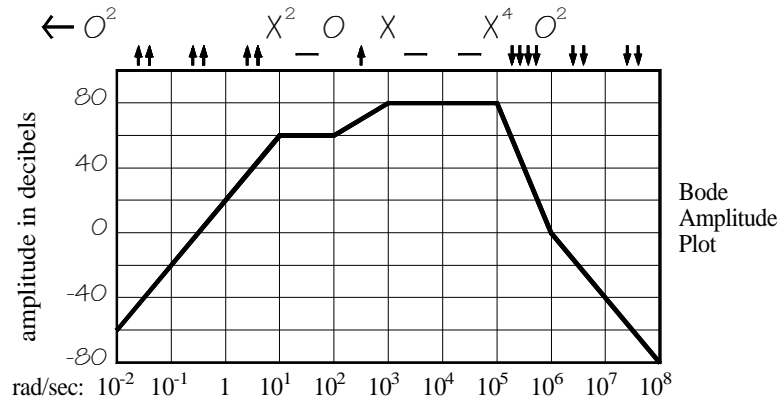
Determining the amplitudes for the Bode Amplitude Plot:

Label the -60 dB starting point at $\omega = 10^{-2}$. For the two up arrows in the first decade, add 40 dB and label the -20 dB result at $\omega = 10^{-1}$. Continue across the plot, adding 20 dB for each up arrow and subtracting 20 dB for each down arrow.



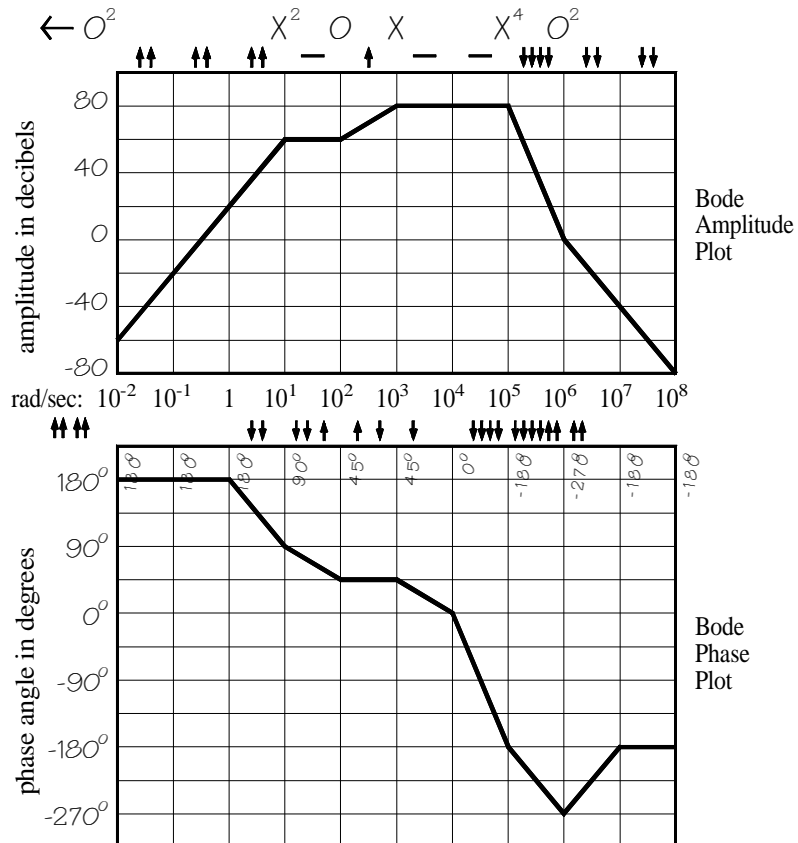
Drawing the Bode Amplitude Plot:

We now have all the information we need to label the vertical axis and draw the Bode Phase Plot.



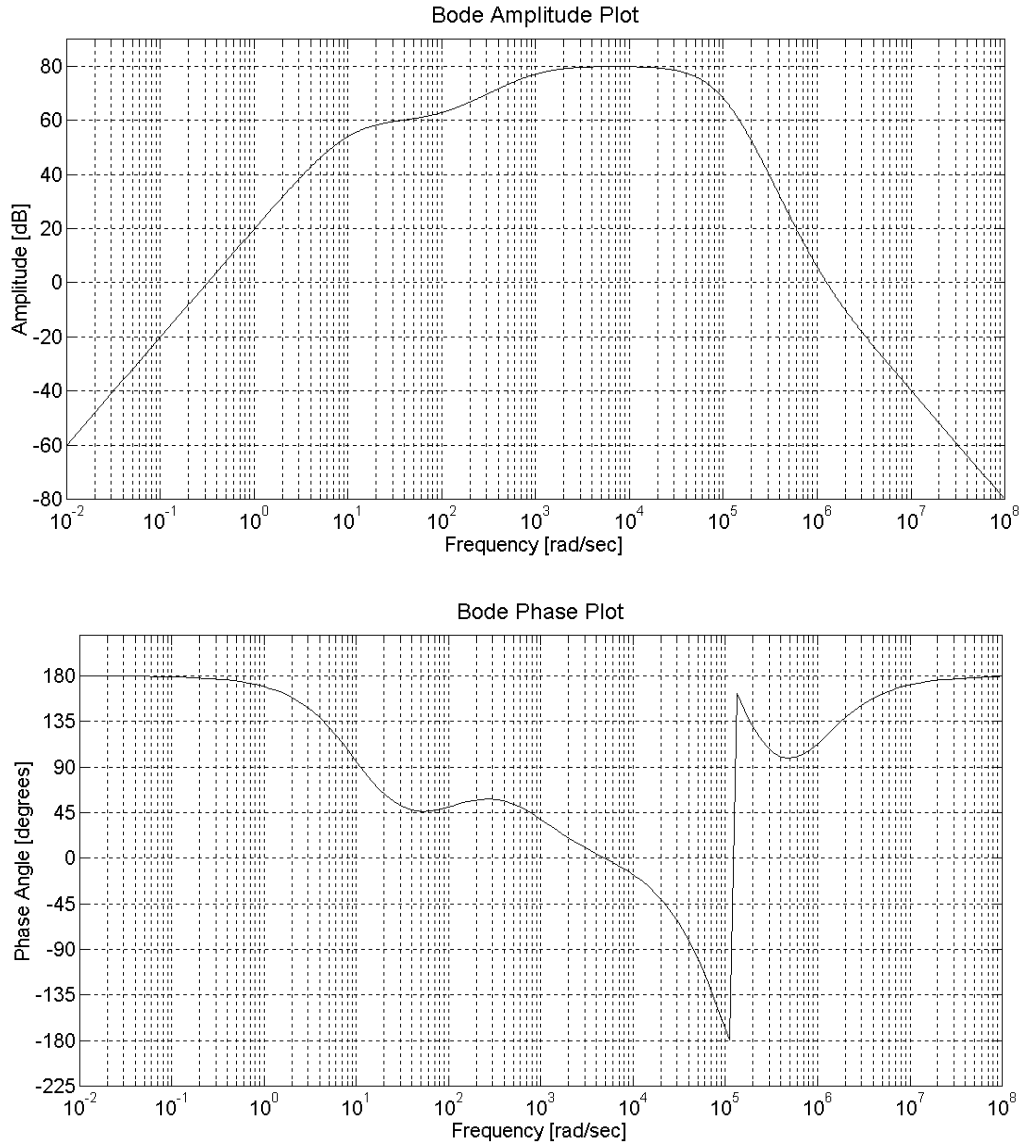
The Results:

The resulting Bode Amplitude and Phase Plots are approximations. The actual plots (see next page) would feature curved transitions at the *frequency corners* rather than the abrupt changes in slope seen in these plots.



The Bode Plots done in Matlab:

This is the same function plotted in Matlab. The abrupt vertical shift in the Phase Plot is an error due to the function passing the -180° mark.



The Matlab Code:

```
% This function creates two bode plots (amplitude and phase) for a transfer function.
% You must edit this file under "*** THE EQUATION: ***" and enter the function y(s).
% There are some sample functions below that can be copied and pasted into the
% proper location. Then, to make it happen, just type the name of this file in the
% Matlab window and press <Enter>. You can change the size of the plot window by
% editing the values of the PlotWindowSize variable.

% Problem 1: y = 10*s^2 * (1+s/10^2) * (1+s/10^6)^2 / ((1+s/10)^2 * (1+s/10^3) * (1+s/10^5)^4);
% Problem 2: y = 10^10 * (1+s/10^2) * (s+10^5) / (s*(s+10^3) * (s+10^4)^2);
% Problem 3: y = 10^26*s*(s+1)*(s+10^3)*(s+10^5)^2 / ((s+10)^2*(s+10^2)*(s+10^4)*(s+10^6)^5);
% Problem 4: y = (s+.1) * (s+1)^3 * (s+10^4)^2 / (10^5*s^2 * (1+s/10)^3 * (1+s/10^3)^2);
% Problem 5: y = 10^10*s * (s+.1)^2 * (s+10^2)^3 / ((s+1) * (s+10)^4 * (1+s/10^4)^2 * (s+10^5)^2);

PlotWindowSize = [1400,600]; % plot window size in pixels
figure('Position',[10 10 PlotWindowSize(1) PlotWindowSize(2)])
N=[]; X=[]; Y=[]; x=10^-2; % initialize variables
while x < 10^8, % repeat for values of x start : increment : end
s = i*x; % get the j-omega

% ***** THE EQUATION: *****

y = y = 10*s^2 * (1+s/10^2) * (1+s/10^6)^2 / ((1+s/10)^2 * (1+s/10^3) * (1+s/10^5)^4);

X = [X,x]; N = [N,y]; % form matrices for X and complex Y values
x = x*1.05; % controls the resolution of the plots
end % end of "while" statements

% ***** THE AMPLITUDE PLOT *****
Y = 20*log10(abs(N)); % get the Y-matrix in dB
semilogx(X,Y); % variables to plot
set(gca,'FontSize',16)
grid on % make the grid visible
xlims = get(gca,'Xlim');
ylims = get(gca,'Ylim');
ylims = [ylims(1),ylims(2)+10]; % add 10 dB headroom to the graph
set(gca,'Ylim',ylims); % set the new y-limit
yinc = 20; % y-axis increments are 20 dB
set(gca,'Xtick',[10^-2 10^-1 10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8])
set(gca,'Ytick',[ylims(1):yinc:ylims(2)])

title('Bode Amplitude Plot','FontSize',18,'Color',[0 0 0])
xlabel('Frequency [rad/sec'],'FontSize',16, 'Color',[0 0 0])
ylabel('Amplitude [dB]','FontSize',16, 'Color',[0 0 0])

% ***** THE PHASE PLOT *****
figure('Position',[10 60 PlotWindowSize(1) PlotWindowSize(2)])
Y = angle(N)*180/pi; % get the angles of the complex matrix in degrees

semilogx(X,Y); % variables to plot
set(gca,'FontSize',16)
grid on % make the grid visible
xlims = get(gca,'Xlim');
ylims = get(gca,'Ylim');
testY = fix(ylims(1)/45); % series of steps to make y-axis label values
if testY < 0 % to be multiples of 45 degrees.
testY = testY - 1;
else
testY = testY + 1;
end
ylims = [testY*45,ylims(2)+20]; % add 10 dB headroom to the graph
set(gca,'Ylim',ylims); % set the new y-limit
yinc = 45; % y-axis increments are 45°
set(gca,'Xtick',[10^-2 10^-1 10^0 10^1 10^2 10^3 10^4 10^5 10^6 10^7 10^8])
set(gca,'Ytick',[ylims(1):yinc:ylims(2)])

title('Bode Phase Plot','FontSize',18,'Color',[0 0 0])
xlabel('Frequency [rad/sec'],'FontSize',16, 'Color',[0 0 0])
ylabel('Phase Angle [degrees]','FontSize',16, 'Color',[0 0 0])
```