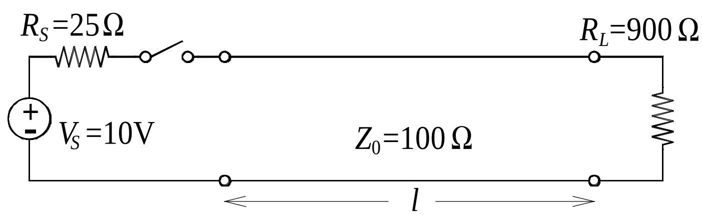
CREATING A BOUNCE DIAGRAM

This document describes creating a bounce diagram for a transmission line circuit.

The Problem:

Given the transmission line circuit:



A 10V DC source with an internal resistance of 25Ω is connected to a transmission line of length *l* having an impedance of 100Ω by a switch. The transmission line is terminated with a 900Ω load resistor. *T* is the amount of time required for a signal to travel the length of the transmission line.

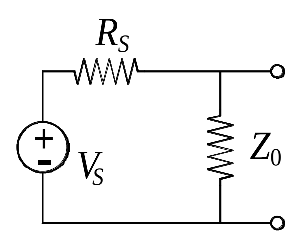
Sequence of Events and Their Results:

1) The switch is closed:

At time t=0, the switch is closed and the source end of the transmission line becomes energized. At this moment, the source *sees* only the impedance of the line. The effects of the load will not be *felt* at the source until t=2T, the time it takes for the signal to travel to the load and back to the source.

The voltage of the initial wave V_{1+} can be found by voltage division using the circuit model at right.

$$V_{1+} = V_S \frac{z_0}{z_0 + R_S} = 10\frac{100}{100+25} = 8 \text{ volts}$$



Circuit model at t=0

The signal proceeds to the end of the transmission line in time T, charging the entire line to a potential of 8 volts.

2) The first reflection (at the load):

When the signal V_{1+} reaches the end of the transmission line and encounters the resistive load, a portion of V_{1+} is reflected back into the line. The portion of V_{1+} that is reflected is determined by the **reflection coefficient** of the load. The load reflection coefficient is calculated as follows:

$$\Gamma_L = \frac{z_L - z_0}{z_L + z_0} = \frac{900-100}{900+100} = \frac{800}{1000} = 0.8$$

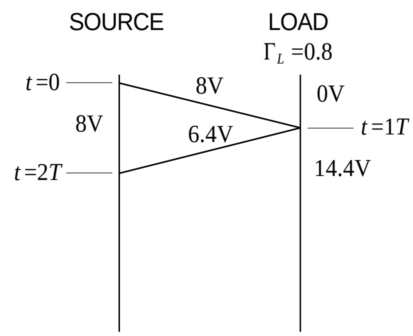
The reflected signal V_{1-} is calculated by multiplying the initial signal by the load reflection coefficient:

$$V_{1-} = V_{1+}(0.8) = 8(0.8) = 6.4 \text{ volts}$$

Following the reflection, the voltage at the load end of the transmission line will be the sum of the initial signal and the reflected signal:

$$V_{total} = V_{1+} + V_{1-} = 8 + 6.4 = 14.4 \text{ volts}$$

As the reflected signal returns to the source end of the line, the entire transmission line is charged to 14.4 volts. The information we have collected thus far can be entered into a **bounce diagram**.



Bounce diagram

3) The second reflection (at the source):

When the signal V_{1-} reaches the source end of the transmission line and encounters the source resistance, a portion of V_{1-} is reflected back into the line. The portion of V_{1+} that is reflected is determined by the **reflection coefficient** of the source. The source reflection coefficient is calculated as follows:

$$\Gamma_S = \frac{z_S - z_0}{z_S + z_0} = \frac{25-100}{25+100} = \frac{-75}{125} = -0.6$$

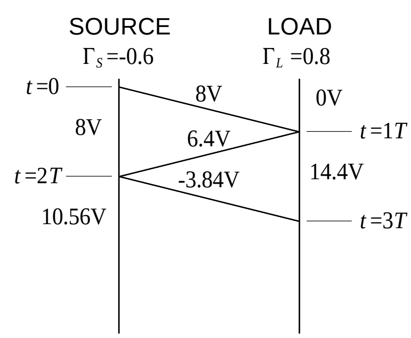
The reflected signal V_{2+} is calculated by multiplying the returning signal V_{1-} by the source reflection coefficient:

$$V_{2+} = V_{1-}(-0.6) = 6.4(-0.6) = -3.84 \text{ volts}$$

Following the reflection, the voltage at the source end of the transmission line will be the sum of the initial signal, the first reflection, and the second reflection:

$$V_{total} = V_{1+} + V_{1-} + V_{2+} = 8 + 6.4 + (-3.84)$$
$$= 10.56 \text{ volts}$$

As the reflected signal travels to the load end of the line, the entire transmission line is charged to 10.56 volts. We add this information to the bounce diagram at right.



Bounce diagram

4) The third reflection (at the load):

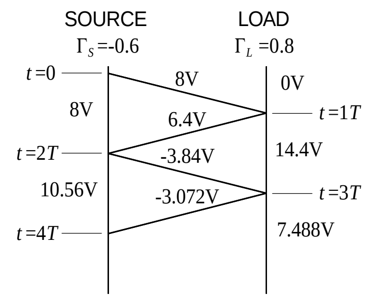
When the signal V_{2+} reaches the end of the transmission line and encounters the resistive load, a portion of V_{2+} is reflected back into the line. The reflected signal V_{2-} is calculated by multiplying the incoming signal by the load reflection coefficient:

$$V_{2-} = V_{2+}(0.8) = -3.84(0.8) = -3.072 \text{ volts}$$

Following the reflection, the voltage at the load end of the transmission line will be the sum of the initial signal and all reflected signals:

$$V_{total} = V_{1+} + V_{1-} + V_{2+} + V_{2-}$$
$$= 8 + 6.4 + (-3.84) + (-3.072) = 7.488 \text{ volts}$$

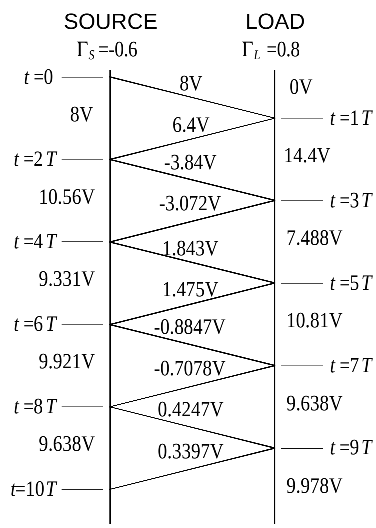
As the reflected signal returns to the source end of the line, the entire transmission line is charged to 7.488 volts. This information is added to the bounce diagram at right.



Bounce diagram

5) More reflections:

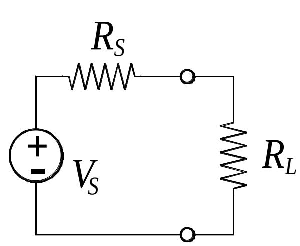
We can continue to add the to bounce diagram in the same fashion. The diagram at right has been carried out to t=9T:



Bounce diagram

The steady-state condition:

It can be seen in the bounce diagram at right that the traveling voltages are becoming smaller and smaller and that the total voltages at the source and load ends of the transmission line are converging to a value between 9.6 and 10 volts. By performing a steady-state analysis, we can find the ultimate voltage which will be present at both ends of the transmission line at t=∞ (assuming the line has no resistance). Using the circuit model shown, we find the steady-state voltage using voltage division:

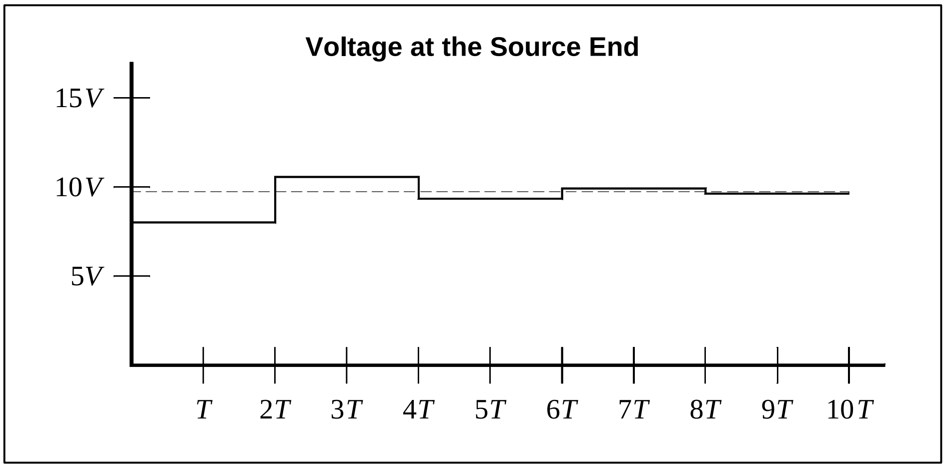


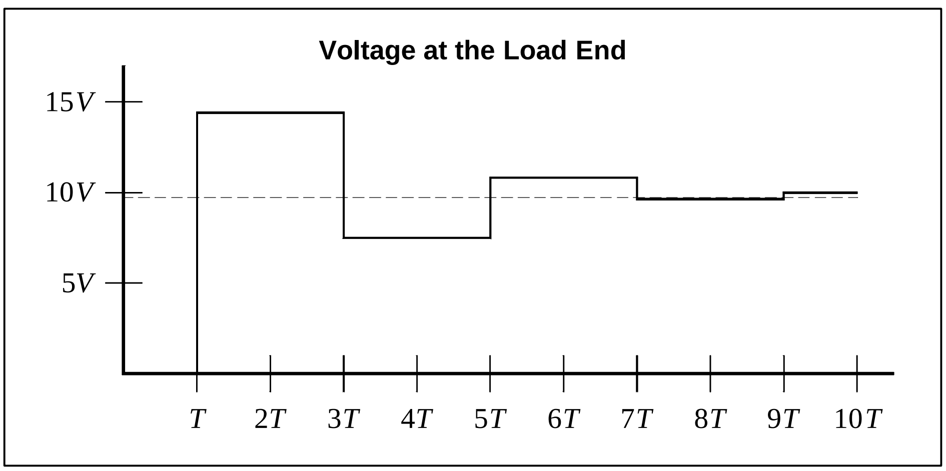
Steady-state circuit

$$V_{SS} = V_S \frac{R_L}{R_L + R_S} = 10 \frac{900}{900 + 25} = 9.730 \text{ volts}$$

Other graphs:

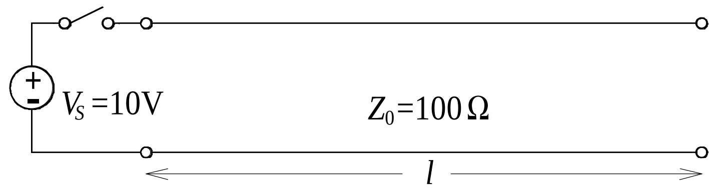
We can also graph the voltage at each end of the transmission line as a function of time. The graphs below are for the previous example problem. The dashed line is the steady-state voltage, 9.730V:





The open-circuit transmission line:

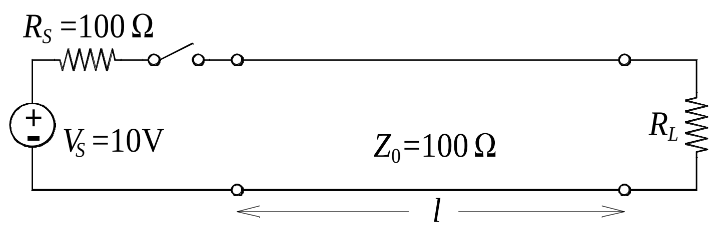
Consider an open circuit transmission line with a perfect DC voltage source (R_L=∞, R_S=0).



In this case, the reflection coefficient at the load end is 1 and the reflection coefficient at the source end is -1. When the switch closes, a 10V signal travels to the load end, is 100% reflected and charges the line to 20V on the return trip. The 10V returning signal is reflected at the source as a –10V signal, thus leaving the source end at a potential of 10V and bringing the load end back to 0V after being reflected at the load. The result is that the source end is always at 10V and the load end experiences a square-wave signal of period 4*T* that is alternately 0V and 20V.

The "matched" transmission line:

Consider a transmission line with a characteristic impedance of 100Ω connected to a DC voltage source with an internal resistance of 100Ω. The line is connected to a load R_L.



In this case, the initial voltage signal will be 5V due to voltage division calculated with R_S and Z_0. The voltage will be partially reflected at the load and will return to the source, charging the line to 5V+V_{1-}. The reflection coefficient at the source is:

$$\Gamma_S = \frac{z_S - z_0}{z_S + z_0} = \frac{100-100}{100+100} = \frac{0}{200} = 0$$

In other words, there is no reflection at the source and therefore no further traveling voltages occur and the line voltage remains constant at 5V+V_{1-}. This is a matched transmission line having the advantage of quickly reaching its working voltage level with minimal voltage excursions.