DIFFERENTIAL EQUATIONS DEFINITIONS

A Glossary of Terms

- **differential equation -** An equation relating an unknown function and one or more of its derivatives
- **first order -** A first order differential equation contains no derivatives other than the first derivative. The order of a differential equation should give the number of constants in the general solution.
- **ordinary -** An ordinary differential equation has only one independent variable
- **autonomous** Describes a differential equation in which the independent variable *t* does not appear explicitly, for example, $\frac{dx}{dt} = 4x x^2$
- **initial condition -** A specified starting point such as $t_0 = 0$ which represents a point on the graph from which the solution curve begins
- **solution -** A solution for a differential equation is a function whose elements and derivatives may be substituted into the differential equation
- **general solution** The general solution of a differential equation contains an arbitrary constant *C*.
- **particular solution -** When an *initial value* is specified, a solution (function) containing no constant
- trivial solution All coefficients are equal to zero
- **isocline -** A line or curve formed by plotting constant values of the first differential, i.e. points at which the slope is uniform. This is not necessarily a solution of the differential equation. p20
- **separable** A differential equation in which the dependent and independent variables can be algebraically separated on opposite sides of the equation.
- **explicit** An explicit solution of a differential equation is one in which *y* may be written in terms of *x* only. Otherwise it is an **implicit** solution.
- **singular solution -** A singular solution of a differential equation is a *particular solution* which cannot be found by substituting a value for *C*. There may be one or several singular solutions for a differential equation. p31
- **linear** A linear first-order differential equation does not contain a *y* raised to a power other than 1, has no *singular solutions*. The graph of a linear differential is not as busy or odd-looking as the graph of a non-linear equation.
- **integrating factor** A function by which a differential equation is multiplied so that each side may be recognized as a derivative (and then be integrated). p41
- **homogeneous** A differential equation that can be written in the form y' = F(y/x). It is said to be *scaleable*. By substituting for y/x the equation may be made *separable*.
- **Bernoulli** A first-order differential equation that can be written in the form $dy/dx + P(x)y = Q(x)y^n$. If n = 0 or 1 then the equation is *linear*.
- **exact** An exact equation can be written in the form M(x,y)dx+ N(x,y)dy = 0 where the partial derivative of M with

respect to y is equal to the partial derivative of N with respect to x.

analytic - In terms of a power series means converging, having a non-zero denominator

GRAPHING TERMS

- **periodicity** the repetition of a pattern of a plot along the *x*-axis when *x* represents time.
- **damping** The tendency of a plot to collapse to a point. With high damping the plot collapses quickly; with low damping there is a gentle spiral.
- **unstable critical point -** a point which the plot approaches and then veers away from, or a point which anchors an outward spiral
- **stable critical point -** a critical point surrounded by closed trajectories (like circles for example) or a critical point within an inward-moving spiral, the latter being **asymptotically stable**
- **linear plot** has only one center of activity; generally not a complex plot
- **linear system -** has only one critical point which will be located at 0,0
- **non linear plot** has multiple centers of activity, a more complex plot
- **critical point -** a point obtained by setting the derivatives to zero. The graph disappears or collapses at this point--there is no movement (with repect to time).
- **change of variables -** method of transposing a graph so that a critical point is moved to the origin so that its behavior may be more easily analyzed. Given a critical point of (a, b), substitute u + a and v + b for x and y in the equation to transform the critical point to 0, 0.
- **boundary curve -** a curve which is approached but not crossed by the plot. Same as **asymptotic curve**.
- **period** the period of a system can be observed by plotting x and/or y versus t.
- **pure imaginary eigenvalue -** the situation in which l = 0, the plot is closed loops. If the equation is linear, the plot will be circles.
- **spiraling** given the eigenvalues $a \pm ib$, spiraling occurs if a is not zero. If a is positive it means an expanding (outward) spiral; if negative it is a collapsing spiral.

distinct eigenvalues -

- **phase plane -** the *x*-*y* plane of a system of equations expressed as differentials with respect to *t*
- **phase portrait -** a phase plane picture (*x*-*y* graph) of a system of differential equations showing the critical points and typical trajectories
- **saddle point -** the type of critical point obtained when the equation has real roots of opposite sign. Two solution curves cross at this point and asymptotes to nearby solutions
- singularity refers to some sort of "blowup"

equilibrium - is like a point of convergence

- **roots of a characteristic equation -** values of *r* used in some methods to find solutions of second order differential equations where *r* belongs to the quadratic equation $ar^2 + br + c = 0$ and the coefficient of *y*" equals the coefficient of r^2 , the coefficient of *y*' equals the coefficient of *r*, and the coefficient of *y* is the constant. Types of roots are: **pure imaginary -** evidenced by circles or ovals around a critical point
 - **complex conjugates -** evidenced by a critical point anchoring a spiral
 - **real and equal -** multiple lines intersect at the critical point
 - **real and opposite sign -** evidenced by a saddle where two and only two asymptotic curves intersect
 - **real, unequal, same sign -** evidenced by many curves intersecting at a critical point

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