CHARACTERISTIC EQUATIONS

Methods for determining the roots, characteristic equation and general solution used in solving second order constant coefficient differential equations

There are three types of roots, Distinct, Repeated and Complex, which determine which of the three types of general solutions is used in solving a problem.

Distinct Real Roots	Repeated Roots	Complex Roots
If the roots have opposite sign, the graph will be have a **saddle point** where only two asymptotic curves intersect. If the roots are unequal with the same sign, there are many curves intersecting at a critical point.	If the roots are real and equal, the graph of the equation will have **multiple curves** that intersect at a critical point.	If the roots are pure imaginary, the graph will have **circles or ovals** around a critical point. If the roots are complex conjugates, the graph will have a critical point anchoring a **spiral**. If the real part of the conjugate is positive, the spiral is expanding (direction of movement is outward); negative means a collapsing spiral.
Example: $y''+2y'-3y=0$	Example: $y''+4y'+4y=0$	Example: $y''+2y'+5y=0$
Characteristic equation: $r^2+2r-3=0$	Characteristic equation: $r^2+4r+4=0$	Characteristic equation: $r^2+2r+5=0$
which factors to: $(r+3)(r-1)=0$	which factors to: $(r+2)^2=0$	using the quadratic formula: $r=\frac{-2\pm\sqrt{4-20}}{2}$
yielding the roots: $r=-3,1$	yielding the roots: $r=2,2$	yielding the roots: $r=-1\pm 2i$
The formula: $y(x)=c_1e^{r_1x}+c_2e^{r_2x}$	The formula: $y(x)=c_1e^{r_1x}+c_2xe^{r_2x}$	The formula: where $r=a\pm bi$ $y(x)=e^{ax}(c_1\cos bx+c_2\sin bx)$
The general solution is: $y(x)=c_1e^{-3x}+c_2e^{1x}$	The general solution is: $y(x)=c_1e^{2x}+c_2xe^{2x}$	The general solution is: $y(x)=e^{-1x}(c_1\cos 2x+c_2\sin 2x)$

These examples are all homogeneous equations (equal to zero) but the general solutions are also used in solving non-homogeneous equations and are found in the same way, ignoring the non-zero value on the right hand side of the equation.

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