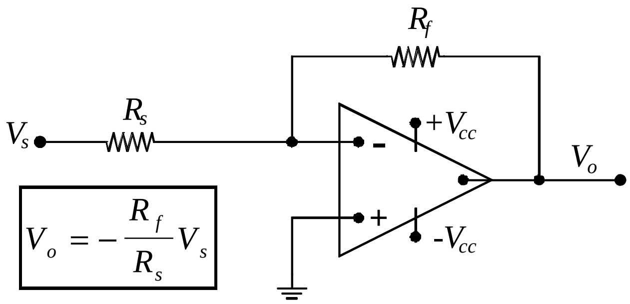
CIRCUIT THEORY EE411

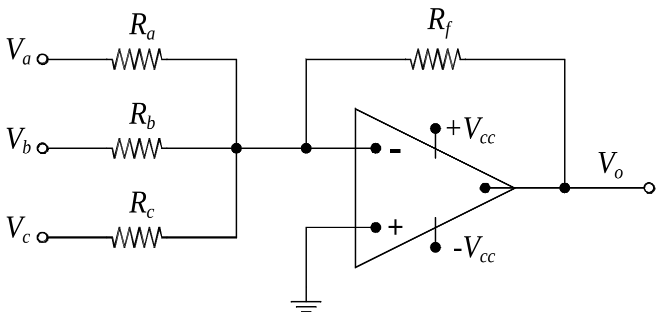
Op Amps

INVERTING AMPLIFIER



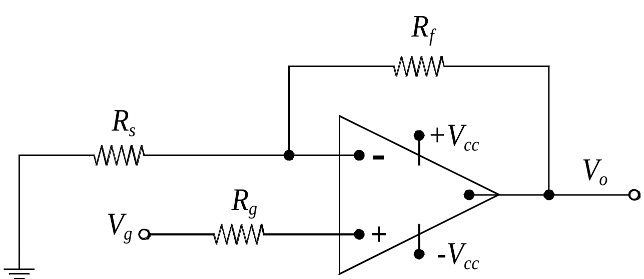
$$V_o = -\frac{R_f}{R_s} V_s$$

INVERTING SUMMING AMPLIFIER



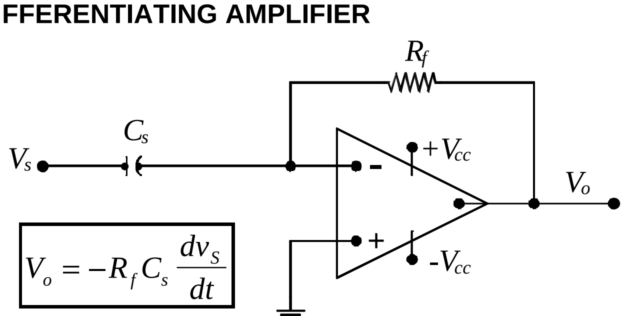
$$V_o = -\left(\frac{R_f}{R_a} V_a + \frac{R_f}{R_b} V_b + \frac{R_f}{R_c} V_c \right)$$

NONINVERTING AMPLIFIER



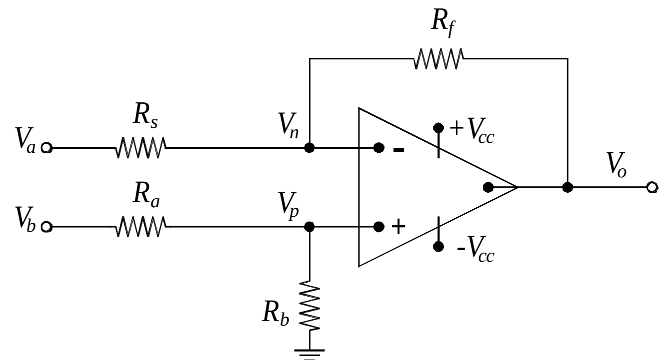
$$V_o = \frac{R_s + R_f}{R_s} V_g$$

DIFFERENTIATING AMPLIFIER



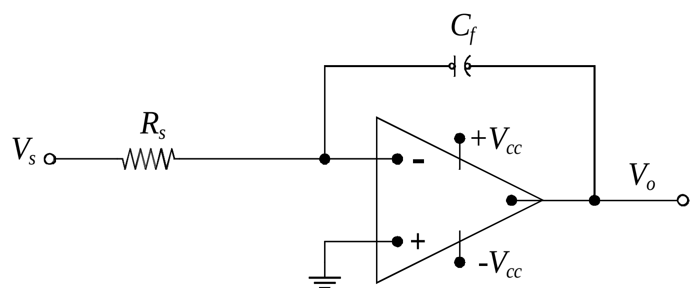
$$V_o = -R_f C_s \frac{dv_S}{dt}$$

DIFFERENCE AMPLIFIER



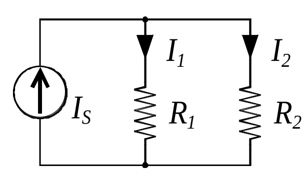
$$V_n = V_p = V_b \frac{R_b}{R_a + R_b} \qquad \frac{V_n - V_a}{R_s} + \frac{V_n - V_o}{R_f} = 0$$

INTEGRATING AMPLIFIER



$$V_o = -\frac{1}{R_s C_f} \int_{t_o}^{t} V_s \tau \, d\tau + V_o(t_o)$$

CURRENT DIVISION



$$I_1 = \frac{R_2}{R_1 + R_2} I_S$$

$$I_2 = \frac{R_1}{R_1 + R_2} I_S$$

THÉVENIN AND NORTON EQUIVALENTS

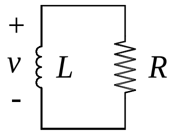
The Thévenin resistance is the resistance in the circuit when voltage sources are shorted and current sources are opened (no independent sources present), *or*

$$R_{TH} = \frac{\text{open circuit voltage}}{\text{short circuit current}}$$

If dependent sources are present, deactivate independent sources by shorting voltage supplies and opening current supplies, and apply a test source. Dependent sources become dependent on the test source. Then

$$R_{TH} = \frac{V_T}{i_T} \qquad I_N = \text{short circuit current}$$

LC CIRCUITS



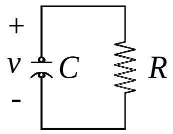
Energy (*joules*): $w = \frac{1}{2} L i^2$

also: $w = \frac{1}{2} L I_o^{\,2} (1 - e^{-2t/\tau})$

Time Constant: $\tau = L / R$

Voltage: $v_L(t) = L \frac{di}{dt}$

Current: $I_L(t) = \frac{1}{L} \int_0^t v \, d\tau + I_o$



Power:
$P = C v \frac{dv}{dt}$

Energy (*joules*): $w = \frac{1}{2} C e^2$

also: $w = \frac{1}{2} C V_o^{\,2} (1 - e^{-2t/\tau})$

Time Constant: $\tau = RC$

Voltage: $V_c(t) = \frac{1}{C} \int_0^t i \, d\tau + V_o$

Current: $i_c(t) = C \frac{dv}{dt}$

Equations Common to L & C Circuits

Current: $i(t) = I_f + (I_o - I_f) e^{-t/\tau}$

Voltage: $v(t) = V_f + (V_o - V_f) e^{-t/\tau}$

Power: $p = I_o^{\,2} R e^{-2t/\tau}$

RLC CIRCUITS -- Parallel

Sum of node currents in a Parallel RLC circuit:

$$C \frac{dv}{dt} + \frac{v}{R} + \frac{1}{L} \int_0^t v \, d\tau + I_o = 0$$

which differentiates to:

$$C \frac{d^2 v}{dt^2} + \frac{1}{R} \frac{dv}{dt} + \frac{1}{L} v = 0$$

RLC CIRCUITS -- Series

Sum of voltages in a Series RLC circuit:

$$L \frac{di}{dt} + Ri + \frac{1}{C} \int_0^t i \, d\tau + V_o = 0$$

which differentiates to:

$$L \frac{d^2 i}{dt^2} + R \frac{di}{dt} + \frac{1}{C} i = 0$$

$$\frac{dx}{dt} \Leftrightarrow j\omega \mathrm{X} \qquad \frac{d^2 x}{dt^2} \Leftrightarrow (j\omega)^2 \mathrm{X}$$

RLC CIRCUITS – solving second order equations

α the Neper frequency (damping coefficient) [rad/s]:

Parallel circuits: $\alpha = \frac{1}{2RC}$	Series circuits: $\alpha = \frac{R}{2L}$

ω the Resonant frequency [rad/s]:

$\omega_o = \frac{1}{\sqrt{LC}}$	$\omega_d = \sqrt{\omega_o^{\,2} - \alpha^2}$	used in underdamped calculations

s_1 , s_2 the roots of the characteristic equation [rad/s]:

$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_o^{\,2}}$	$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_o^{\,2}}$

RLC CIRCUITS – solving second order equations

Overdamped $\alpha^2 > \omega^2$ (real and distinct roots)

$$X(t) = X_f + A_1' e^{s_1 t} + A_2' e^{s_2 t}$$

$$X(0) = X_f + A_1' + A_2' \qquad \frac{dx}{dt}(0) = s_1 A_1' + s_2 A_2'$$

Underdamped $\alpha^2 < \omega^2$ (complex roots)

$$X(t) = X_f + B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t$$

$$X(0) = X_f + B_1' \qquad \frac{dx}{dt}(0) = -\alpha B_1' + w_d B_2'$$

Critically Damped $\alpha^2 = \omega^2$ (repeated roots)

$$X(t) = X_f + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}$$

$$X(0) = X_f + D_2' \qquad \frac{dx}{dt}(0) = D_1' - \alpha D_2'$$

In an **overdamped** circuit, $\alpha^2 > \omega^2$ and the voltage or current approaches its final value without oscillation.

In an **underdamped** circuit, $\alpha^2 < \omega^2$ and the voltage or current oscillates about its final value.

In a **critically damped** circuit, $\alpha^2 = \omega^2$ and the voltage or current is on the verge of oscillating about its final value.

When an expression is integrated, it may be necessary to add in initial values for the constant of integration even if they have been taken into account within other terms.

Natural response is the behavior of a circuit without external sources of excitation.

Step response is the behavior of a circuit with an external source.

Trig Identities

$$A \cos \omega t + B \sin \omega t = \sqrt{A^2 + B^2} \cos\left[\cot + \tan\left(\frac{-B}{A}\right)\right]$$

$c^{\pm j\theta} = \cos\theta \pm j \sin\theta$ Euler identity

$\sin \omega t = \cos(\omega t - 90^\circ)$

SINUSOIDAL ANALYSIS

$$\pi \times \text{degrees} = 180 \times \text{radians}$$

$$\omega = 2\pi f \ [\text{rad/s}] = 360 f \ [\text{deg/s}]$$

resonant frequency $\omega_o = \frac{1}{\sqrt{LC}}$

$v(t) = V_m \cos(\omega t + \phi) \qquad i(t) = I_m \cos(\omega t + \phi)$

where V_m and I_m are maximums

$$V_{rms} = \frac{V_{\max}}{\sqrt{2}} \qquad X_{rms} = \sqrt{\frac{1}{T} \int_0^T \left(f(x)\right)^2 dt}$$

equivalent of two parallel impedances $= \frac{\text{product}}{\text{sum}}$

Phasor Transform:	$\mathrm{V} = V_m e^{j\phi} = \mathrm{P}\{V_m \cos(\omega t + \phi)\}$ $v(t) = A \cos(\omega t + \phi^\circ) \Leftrightarrow A \angle \phi^\circ$ $\sin \omega t = \cos(\omega t - 90^\circ)$
Inverse Phasor Transform	$\mathrm{P}^{-1}\{V_m e^{j\phi}\} = \mathrm{R}\{V_m e^{j\phi} e^{j\omega t}\}$

A smaller ϕ causes a right shift of the sinusoidal graph.

SINUSOIDAL ANALYSIS

Element:	Resistor	Capacitor	Inductor
Impedance (Z):	R (resistance)	$-j/\omega C$	$j\omega L$
Reactance (X)	--	$-1/\omega C$	ωL
Admittance (Y):	G (conductance)	$j\omega C$	$1/j\omega L$
Susceptance:	--	ωC	$-1/\omega L$
Voltage:	$\mathbf{I}R$	$\mathrm{I}/j\omega C$ $(I_m/\omega C)\angle(\theta_V - 90°)$	$j\omega L\mathrm{I}$ $\omega L I_m \angle(\theta_V + 90°)$
Amperage:	$\mathbf{V}/R$	$j\omega C\mathrm{V}$ $(V_m/\omega C)\angle(\theta_V + 90°)$	$\mathrm{V}/j\omega L$ $(V_m/\omega L)\angle(\theta_V - 90°)$

PHASOR and RECTANGULAR NOTATION

To convert from **rectangular to phasor** notation:

Rectangular form: $X \pm jY$

Magnitude: $M = \sqrt{X^2 + Y^2}$

Angle f: $\tan\phi = \dfrac{Y}{X}$ (Caution: The Y will be negative is the **j** value is being subtracted from the real.)

Note: Due to the way the calculator works, if X is negative, you must **add 180°** after taking the inverse tangent. If the result is greater than 180°, you may optionally subtract 360° to obtain the value closest to the reference angle.

To convert from **phasor to rectangular** (j) notation:

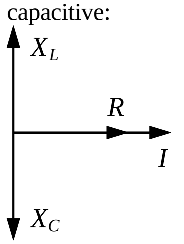
Phasor form: $M\angle\phi°$

X (real) Value: $M\cos\phi$

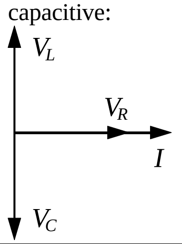
Y (**j** or imaginary) Value: $M\sin\phi$

In conversions, the j value will have the same sign as the θ value for angles having a magnitude < 180°.

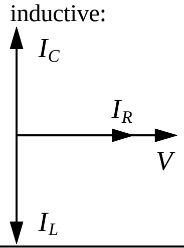
Phasor impedance diagram for series circuits, where +**j** is inductive and -**j** is capacitive:



Phasor voltage diagram for series circuits, where +**j** is inductive and -**j** is capacitive:



Phasor amperage diagram for parallel circuits, where +**j** is capacitive and -**j** is inductive:



POWER

Average Power or real power (*watts*)

$$P = \frac{V_m I_m}{2}\cos(\theta_v - \theta_i)$$
$$= V_{rms} I_{rms}\cos(\theta_v - \theta_i)$$

Positive P means absorbing **average power,** negative means delivering or generating.

Reactive Power (*VARS*)

$$Q = \frac{V_m I_m}{2}\sin(\theta_v - \theta_i)$$
$$= V_{rms} I_{rms}\sin(\theta_v - \theta_i)$$

Positive Q means absorbing **magnetizing vars** (inductive), negative means delivering (capacitive).

Complex Power (*VA*)

$$\mathrm{S} = P + jQ$$
$$= V_{rms} I_{rms}(\theta_v - \theta_i)$$
$$= \mathbf{V}_{rms}\mathbf{I}^*_{rms} = \frac{1}{2}\mathbf{V}_{\max}\mathbf{I}^*_{\max} = \frac{\mathbf{V}^2_{rms}}{\mathbf{Z}^*}$$

* means "the complex conjugate of"

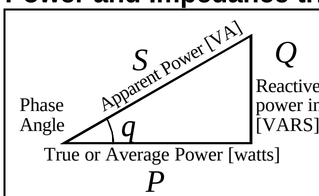
Power Factor (ratio of true power to apparent power)

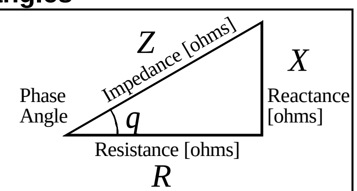
$pf = \cos(\theta_v - \theta_i)$

Lagging: Inductive, current lags (-j), +Q

Leading: Capacitive, current leads (+j), -Q

Power and Impedance triangles





$\theta = \cos^{-1} pf$ The power triangle is geometrically identical to the impedance triangle.

Maximum Power Transfer

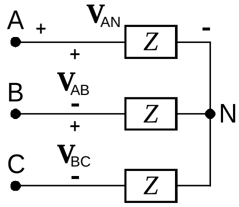
Maximum power transfer occurs when the load impedance is equal to the complex conjugate of the source impedance. Under these conditions, the maximum value of average power absorbed is

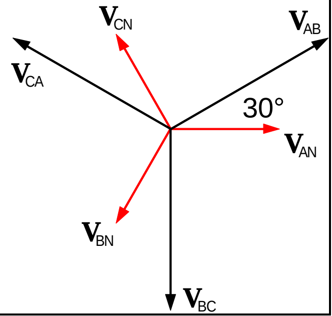
$$P_{\max} = \frac{|V_{TH}|^2}{4R_L}$$

3-PHASE POWER

Phase and line voltage relationships in a Wye Circuit

(positive sequence - clockwise)

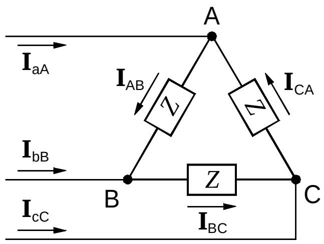


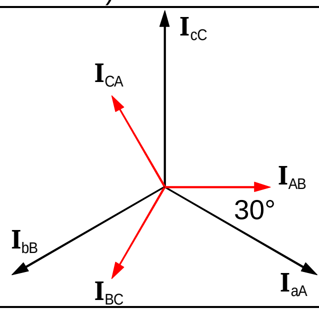


$V_{AB} = V_{AN}\sqrt{3}$ or

$\mathbf{V}_{AB} = \mathbf{V}_{AN}\sqrt{3}\times(1\angle 30^\circ)$

Phase and line current relationships in a Delta Circuit (positive sequence - clockwise)





$I_{aA} = I_{AB}\sqrt{3}$

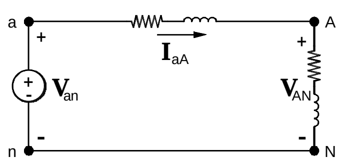
Wye-Delta Transform (for balanced circuits only)

$$Z_Y = \frac{Z_D}{3}$$

$$V_{line-to-line} = V_{an}\sqrt{3}$$

$$I_{aA} = \frac{V_{an}}{Z_f}$$

Single-phase Equivalent Circuit



Motor Ratings

$$P = \sqrt{3}V_L I_L \cos(\theta_v - \theta_i) = \frac{hp \times 746}{\text{efficiency}}$$ where:

P is the power input in watts

$\cos(\theta_v - \theta_i)$ is the power factor

efficiency is expressed as a decimal value

Power Factor Correction

$$Q = \frac{\text{VARS}}{3} = \frac{(460/\sqrt{3})^2}{x_c = -1/\omega C}$$ where:

VARS is a negative value for the amount of correction

460 is the line voltage

C is the value of the capacitor in Farads

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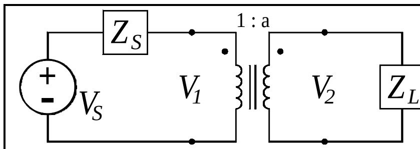
TRANSFORMERS

Ideal Transformer

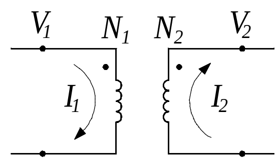


$$Z_{IN} = \frac{V_1}{I_1} = \frac{1}{a^2}\frac{V_2}{I_2} = \frac{1}{a^2}Z_L$$

$$a = \frac{N_1}{N_2} \qquad V_1 = \frac{V_2}{a} \qquad I_1 = aI_2$$

Z_{IN} is the load seen by the source.

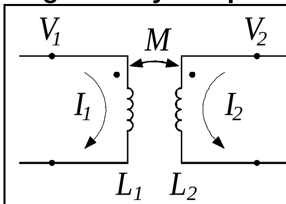
Transformer Turns Ratio



$$\frac{V_1}{N_1} = \frac{V_2}{N_2} \qquad N_1\mathbf{I}_1 = N_2\mathbf{I}_2$$

Moving a dot results in a negative sign on one side of each equation.

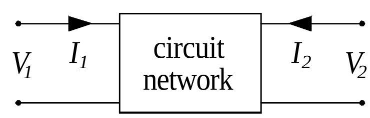
Magnetically Coupled Coils



$$V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$

$$V_2 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

TWO-PORT CIRCUITS



$$V_1 = z_{11}I_1 + z_{22}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$
$$V_1 = z_{12}I_2 + z_{11}I_1 ?$$

$$z = \frac{1}{y}$$

$$I_1 = y_{11}V_1 + y_{22}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$
$$I_2 = y_{12}V_2 + y_{11}V_1 ?$$

$$V_1 = h_{11}I_1 + h_{22}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$
$$I_1 = h_{12}I_2 + h_{11}V_1 ?$$

h parameters are used for transistor specifications

y parameters may be easier to find than z parameters and may be added when networks are paralleled.

Mesh Current Equations involving mutual inductors

A mesh is a loop that does not enclose other loops in the circuit.

1. Draw current loops emanating from positive voltage sources if present and label I_1, I_2, I_3, etc. for each interior path of the circuit.
2. For each loop form an equation in the form: Voltage or 0 if there is no source in the loop = R_1 × (sum of amperages passing through R_1) + $L_1 \frac{d}{dt}$ × (sum of amperages passing through L_1) + . . .
3. Amperages are positive in the direction of loops regardless of the location of dots on inductors in the loop. However, the sign of an amperage through a mutual inductor is positive iff it enters the mutual inductor at the same end (i.e. dotted or undotted) at which the reference current loop enters the reference inductor.