EXPONENTIAL AND LOGARITHMIC FUNCTIONS
Chapter 7

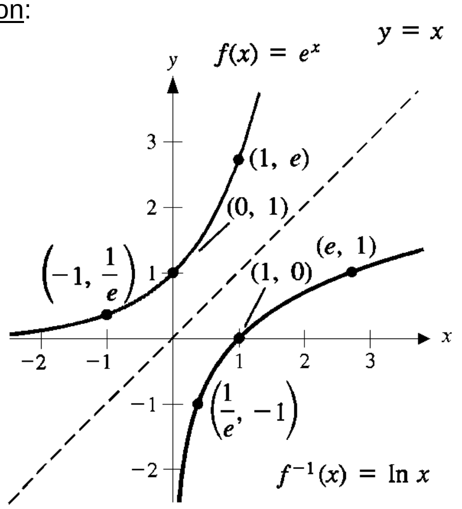
Definition of **Exponential Function**: $f(x) = b^x$ where x and b are real numbers and $b > 0$ and $b \neq 1$. The domain is the set of all real numbers and the range is the set of all positive real numbers.

Exponents: 7.1 p358

$$b^x b^y = b^{x+y} \qquad (b^x)^y = b^{xy} \qquad b^x / b^y = b^{x-y}$$

$b^x > 0$ $\qquad$ $b^x = b^y$ if and only if $x = y$

The logarithm function is the inverse of the exponential function:



The **natural number *e***: 7.1 p360 $e \approx 2.71828182846$ To get this number on the calculator, press `1 INV lnx`

$e = \lim_{x \to 0} (1 + x)^{1/x}$ $\qquad$ $a^x = e^{(\ln a)x}$

$\ln e^x = x$ $\qquad$ $e^{\ln x} = x$

$$\int e^x dx = e^x + C$$

$$\int e^u dx = \frac{1}{u'} \cdot e^u + C \qquad \frac{d}{dx}(e^u) = e^u \cdot u'$$

where x is the variable x and u is a function differentiable function of x.

Definition of **Inverse Function**: 7.3 p371 A function g is the inverse of f if $f(g(x)) = x$ for each x in the domain of g and $g(f(x)) = x$ for each x in the domain of f. We denote g by f^{-1} (read "f inverse") Note that this doesn't mean $1/f(x)$.

In other words, the domain of f equals the range of g and the domain of g equals the range of f. If the graph of f contains the point (a, b), then the graph of g contains the point (b, a). The function of g is a reflection of f about the line $x = y$. To convert f to g, just exchange the variables x and y.

A function possesses an inverse if and only if it is one-to-one. In other words, a *horizontal line* intersects the graph at most once.

The Derivative of an Inverse Function: If f possesses an inverse function g then:

$$g'(x) = \frac{1}{f'(g(x))} \qquad f'(g(x)) \neq 0$$

Definition of the **Natural Logarithmic Function**: 7.4 p379

$$f(x) = \ln x$$

$\log_e x$ is written $\ln x$ (read "el - en - ex")

$\ln x = b$ if and only if $e^b = x$

The domain is $(0, \infty)$, and the range is $(-\infty, \infty)$.

The function is continuous, increasing, and one-to-one on its entire domain.

The graph is concave downward on its entire domain.

The y-axis is a vertical asymptote of its graph.

$\lim_{x \to 0^+} \ln x = -\infty$ $\qquad$ $\lim_{x \to \infty} \ln x = \infty$.

$\ln e^x = x$ $\qquad$ $e^{\ln x} = x$

$\ln xy = \ln x + \ln y$ $\qquad$ $\ln \frac{x}{y} = \ln x - \ln y$

$\ln x^y = y \ln x$ $\qquad$ $\frac{d}{dx} \ln|u| = \frac{u'}{u}$

$\frac{d}{dx}(\ln x) = \frac{1}{x}, \quad x > 0$

$$\boxed{\frac{d}{dx}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx} = \frac{u'}{u}, \quad u > 0}$$

$\frac{d}{dx}(u \cdot \ln x) = u \cdot \frac{1}{x} + \ln x \cdot u'$

where x is the variable x and u is a differentiable function of x.

To differentiate the function $y = u$: 7.5 p388

Take the natural logarithm of both sides: $\ln y = \ln u$

Use logarithmic properties to rid $\ln u$ of as many products, quotients, and exponents as possible.

Differentiate ***implicitly***: $\frac{y'}{y} = \frac{d}{dx}(\ln u)$

Solve for y': $y' = y\frac{d}{dx}(\ln u)$

Substitute for y and simplify: $y' = u\frac{d}{dx}(\ln u)$

Logarithms to other bases:

$y = \log_a x$ if and only if $a^y = x$

$\log_a xy = \log_a x + \log_a y$ $\qquad \log_a \frac{x}{y} = \log_a x - \log_a y$

$\log_a x^y = y \log_a x$ $\qquad \log_a x = \frac{\log_b x}{\log_b a}$

$\frac{d}{dx}\left(a^x\right) = (\ln a)a^x$ $\qquad \frac{d}{dx}\left(a^u\right) = (\ln a)a^u \cdot u'$

$\frac{d}{dx}\left(\log_a x\right) = \frac{1}{(\ln a)x}$ $\qquad \frac{d}{dx}\left(\log_a u\right) = \frac{1}{(\ln a)u} \cdot u'$

A **calculator** can be used to evaluate an expression such as $\log_2 14$ by virtue of the fact that it is equivalent to $\ln 14 / \ln 2$.

The **Power Rule** for real exponents: 7.5 p391

$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$ $\qquad \frac{d}{dx}\left(u^n\right) = nu^{n-1} \cdot u'$

The **Log Rule** for integration: 7.6 p394

$\int \frac{1}{x}dx = \ln|x| + C$ $\qquad \int \frac{1}{u}du = \ln|u| + C$

also: $\int a^x dx = \left(\frac{1}{\ln a}\right)a^x + C$ $\qquad \int \frac{u'}{u} = \ln|u| + C$

where x is the variable x and u is a function differentiable function of x.

Guidelines for **Integration**: 7.6 p397

When the power in the numerator is ≥ the power in the denominator, do long division.

Available formulas are Power Rule, Exponential Rule, Log Rule, plus those in Chap. 8.

Find a u substitution that makes the problem look like a formula, or try altering the integrand.

L'Hôpital's Rule: 7.8 p407 If the limit of $f(x)/g(x)$ as x approaches c produces the indeterminate form 0/0, ∞/∞, or $-\infty/\infty$, then

$$\lim_{x \to c}\frac{f(x)}{g(x)} = \lim_{x \to c}\frac{f'(x)}{g'(x)}$$

provided the limit on the right exists or is infinite.

To convert a limit to a form on which *L'Hôpital's Rule* can be used, try algebraic manipulation *or* try setting y equal to the limit then take the natural log of both sides. The ln can be placed to the right of lim. This is manipulated into fractional form so L'Hôpital's Rule can be used, thus getting rid of the ln. When this limit is found, this is actually the value of $\ln y$ where y is the value we are looking for.

Other indeterminate forms (which might be convertable) are 1^∞, ∞^0, 0^0, $0 \cdot \infty$, and $\infty - \infty$.

$0^\infty = 0$.

Review of the Rules of Differentiation

The derivative of a **constant** is 0. 3.3 p113

The **power** rule: 3.3 p114 the derivative of x^n is nx^{n-1}.

The **general power** rule: 3.5 p131 where u is a function of x, the derivative of u^n is $nu^{n-1}u'$

The **constant multiple** rule: the derivative of $c \cdot f(x)$ is $c \cdot f'(x)$. 3.3 p115

The **sum and difference** rule: 3.3 p116

$\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

The **product** rule: 3.4 p121

$\frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

The **quotient** rule: 3.4 p124

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

The **chain** rule: 3.5 p129 $\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1} \cdot f'(x)$

The **absolute value** rule: 3.5 p135 $\frac{d}{dx}|u| = u'\frac{u}{|u|}, \quad u \neq 0$

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