Applications of Integration
Chapter 6

<u>Area of a region</u> <u>between two curves</u>: 6.1 p293 If f and g are continuous on $[a, b]$ and $g(x) \le f(x)$ for all x in $[a, b]$, then the area of the region bounded by the graphs of f and g and the vertical lines $x = a$ and $x = b$ is

$$A = \int_a^b [f(x) - g(x)]dx$$

In other words, integrate the value obtained by subtracting the lower curve from the upper curve. For curves that intersect in more than two points, it is necessary to treat each segment separately.

DISK METHOD: Finding the volume of a solid of revolution: 6.2 p303 Integrate the square of the **radius** and multiply by π.

$$V = \pi\int_a^b [f(x)]^2 dx$$ (revolving around the x-axis)

$$V = \pi\int_c^d [f(y)]^2 dy$$ (revolving around the y-axis)

WASHER METHOD: Finding the volume of a solid of revolution: 6.2 p305 Subtract the square of the inner radius from the square of the outer radius, integrate the result and multiply by π.

$$V = \pi\int_a^b ([R(x)]^2 - [r(x)]^2)dx$$ (horiz. axis of revolution)

$$V = \pi\int_c^d ([R(y)]^2 - [r(y)]^2)dy$$ (vert. axis of revolution)

SHELL METHOD: Finding the volume of a solid of revolution: 6.3 p313

$$V = 2\pi\int_c^d p(y)h(y)\,dy$$ (horizontal axis of revolution)

$$V = 2\pi\int_a^b p(x)h(x)\,dx$$ (vertical axis of revolution)

where: $p(x)$ and $p(y)$ are **radii** in terms of ***x*** and ***y*** respectively. (These will simply be x or y where the axis of revolution is the y-axis or x-axis.)

$h(y)$ is the horizontal distance across the area in terms of ***y***.

$h(x)$ is the vertical distance across the area in terms of ***x***.

$[a, b]$ is the interval on the x-axis.

$[c, d]$ is the interval on the y-axis.

Definition of **Arc Length**: 6.4 p321 If $f(x)$ represents a smooth curve on the interval $[a, b]$, then the arc length of f between a and b is given by:

$$S = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$$

Definition of the **Area of a Surface of Revolution**: 6.4 p328 If $f(x)$ has a continuous derivative on the interval $[a, b]$, then the area S of the surface of revolution formed by revolving the graph of f about a horizontal or vertical axis is:

$$S = 2\pi\int_a^b r(x)\sqrt{1 + [f'(x)]^2}\,dx$$ (*horizontal axis*)

or

$$S = 2\pi\int_c^d r(y)\sqrt{1 + [f'(y)]^2}\,dy$$ (*vertical axis*)

where $r(x)$ or $r(y)$ is the distance between the graph of f and the axis of revolution.

<u>Quadratic Equation</u>

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Tom Penick tomzap@eden.com www.teicontrols.com/notes