DIFFERENTIATION

Chapter 3

The **Derivative** of f at x is given by: 3.1 p98

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

The Alternate form of the **Derivative**: 3.1 p100 The derivative of $f(x)$ at $x = c$ is

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

The **derivative** of a function is the **slope** of the line which is tangent to the function at $(c, f(c))$. In other words, the derivative of function f is the slope of the graph of f at $x = c$. A function does **not** have a derivative at a point in which the graph has a **sharp turn** or **vertical tangent**. 3.1 p96

The Position Function p94

Equation for a free-falling object ignoring the effect of air resistance: $s(t) = -16t^2 + v_o t + h_o$

where: $s(t)$ is the height
-16 is !/2 the force of gravity
t is elapsed time
v_o is the initial velocity (positive is up, negative is down)
h_o is the initial height

The **first derivative** of $s(t)$ is the Velocity Function: 3.2 p107 $v(t) = -32t + v_o$

The **second derivative** of $s(t)$ yields the Rate of Acceleration: 3.2 p108 $a(t) = -32$

Rules of Differentiation

The derivative of a **constant** is 0. 3.3 p113

The **power** rule: 3.3 p114 the derivative of x^n is nx^{n-1}.

The **general power** rule: 3.5 p131 where u is a function of x, the derivative of u^n is $nu^{n-1}u'$

The **constant multiple** rule: the derivative of $c \cdot f(x)$ is $c \cdot f'(x)$. 3.3 p115

The **sum and difference** rule: 3.3 p116

$$\tfrac{d}{dx}[f(x) \pm g(x)] = \tfrac{d}{dx}[f(x)] \pm \tfrac{d}{dx}[g(x)]$$

The **product** rule: 3.4 p121

$$\tfrac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot g'(x) + g(x) \cdot f'(x)$$

The **quotient** rule: 3.4 p124

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

The **chain** rule: 3.5 p129 $\tfrac{d}{dx}[f(x)]^n = n[f(x)]^{n-1} \cdot f'(x)$

The **absolute value** rule: 3.5 p135 $\tfrac{d}{dx}|u| = u' \frac{u}{|u|}, \quad u \neq 0$

Implicit Differentiation 3.6 p137

Implicit Differentiation is used when the equation cannot be expressed in terms of y. Take the differential $\left(\tfrac{d}{dx}\right)$ of both sides of the equation. For instance, the derivative of x^2, written $\tfrac{d}{dx}(x^2)$, would be $2x$. The derivative of y^3, written $\tfrac{d}{dx}(y^3)$, would be $3y^2 \tfrac{dy}{dx}$. Collect the $\tfrac{dy}{dx}$ terms on the left side of the equation and solve for $\tfrac{dy}{dx}$.

Related Rates 3.7 p144

1. Assign symbols to all given quantities and *quantities to be determined.* Make a sketch and label the quantities if feasible.

   Note: A rate is the derivative of the quantity which is changed. For example, if a vessel is being filled at a rate of 10 cu. ft. per min., then $\tfrac{dv}{dt} = 10$. If you are trying to find the rate at which the height of the liquid is increasing, then you are looking for $\tfrac{dh}{dt}$.

2. Write an equation involving the variables whose rates of change either are given or are to be determined.
3. Using the Chain Rule, implicitly differentiate both sides of the equation *with respect to time* t. If there are more than two variables involved it may be necessary to express one variable in terms of another.
4. Substitute into the resulting equation all known values for the variables and their rates of change. Then solve for the required rate of change.