Limits and Continuity

Chapter 2

Limits

If $f(x)$ becomes close to a unique number L as x approaches c from both the right and left side, then the **limit** of $f(x)$ as x approaches c is L, denoted by

$$\lim_{x \to c} f(x) = L.$$

"As $x \to c$ from the right" is written $x \to c^+$

"As $x \to c$ from the left" is written $x \to c^-$

Properties of Limits: Suppose b and c are real numbers and n is a positive integer, and $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$. Then:

1) $\lim_{x \to c} \left[b \cdot f(x)\right] = b \cdot L$
2) $\lim_{x \to c} \left[f(x) \pm g(x)\right] = L \pm M$
3) $\lim_{x \to c} \left[f(x) \cdot g(x)\right] = L \cdot M$
4) $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{M}, \ M \neq 0$
5) $\left[f(x)\right]^n = L^n$

If $p(x)$ is a **polynomial function** and c is a real number, then $\lim_{x \to c} p(x) = p(c)$.

If $r(x) = \dfrac{p(x)}{q(x)}$ is a rational function, and c is any real number, then $\lim_{x \to c} r(x) = \dfrac{p(c)}{q(c)}, \ q(c) \neq 0/$

If $c > 0$ and n is a positive integer, then $\lim_{x \to c} \sqrt[n]{x} = \sqrt[n]{c}$.

if $c < 0$ and n is an odd integer, then $\lim_{x \to c} \sqrt[n]{x} = \sqrt[n]{c}$.

If $\lim_{x \to c} g(x) = L$ and $\lim_{x \to L} f(x) = f(L)$, then $\lim_{x \to c} f \circ g = f(L)$.

Methods of evaluating limits:

1) Graph the function.
2) Make a table of values close to $x = c$.
3) Direct substitution in polynomial functions, rational functions, irrational functions, and composition of functions.

Properties of **Infinite Limits**: Suppose c and L are real numbers and $f(x)$ and $g(x)$ are functions such that $\lim_{x \to c} f(x) = \infty$ and $\lim_{x \to c} g(x) = L$, then:

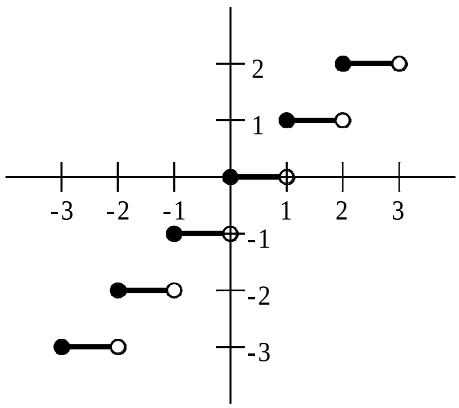
1) $\lim_{x \to c} [f(x) \pm g(x)] = \infty$
2) $\lim_{x \to c} [f(x) \cdot g(x)] = \infty$ if $L > 0$
   and $\lim_{x \to c} [f(x) \cdot g(x)] = -\infty$ if $L < 0$
3) $\lim_{x \to c} \dfrac{g(x)}{f(x)} = 0$

If $\lim_{x \to c^+} f(x) = \pm\infty$ or $\lim_{x \to c^-} f(x) = \pm\infty$, then the vertical line $x = c$ is a **vertical asymptote**. In other words, wherever the y-value goes to infinity, a vertical asymptote exists at the corresponding x-value.

If $h(x) = \dfrac{f(x)}{g(x)}$ and $f(c) \neq 0$ and $g(c) = 0$, then $x = c$ is a **vertical asymptote**. In other words, if the value in the denominator of a function is 0, a vertical asymptote exists at the x-value that causes the denominator to be 0, just as long as the numerator is not 0.

Greatest Integer Function

$y = [\![x]\!] =$ greatest integer n for $n \le x < n + 1$



In other words, if the value within the double brackets is not an integer, then it is rounded down to the next integer.

Continuity

A function is **continuous** at a point $x = c$ if:

1) $f(c)$ is defined
2) $\lim_{x \to c} f(x)$ exists, **and**
3) $\lim_{x \to c} f(x) = f(c)$

A function is **continuous** on an open interval (a, b) if it is continuous at every point in (a, b). A function is **discontinuous** if any of the three conditions above are not met.

Discontinuities are classified into two categories: removable and nonremovable.

A **removable discontinuity** is one in which $f(x)$ could become continuous if $f(x)$ is redefined.

A **nonremovable discontinuity** occurs when $\lim_{x \to a} f(x)$ does not exist.

If $g(x)$ is continuous at $x = c$, and $f(x)$ is continuous at $g(c)$, then the composition $f(g(x))$ is continuous at $x = c$. That is, $\lim_{x \to c} f(g(x)) = f(g(c))$. In other words, the composition $f(g(x))$ is continuous if both $f(x)$ and $g(x)$ are continuous functions.

The **Intermediate Value Theorem**: If $f(x)$ is continuous on $[a, b]$ and k is any number between $f(a)$ and $f(b)$, then there exists a number c in $[a, b]$ such that $f(c) = k$. In other words, if $f(x)$ is a continuous function on $[a, b]$ and if x takes on all values beetween a and b, then $f(x)$ must take on all values between $f(a)$ and $f(b)$.