EXACT EQUATION

Solving an Exact first order differential equation
from section 1.6 p62. See the document FirstOrderDiffEq.pdf
for a comparison between this and other methods.

THE PROBLEM

31. p62 $\boxed{(2x+3y)dx+(3x+2y)dy=0}$ Find the general solution.

The equation is already written in the form $M\ dx + N\ dy = 0$ where $\boxed{\begin{aligned}M &= 2x+3y\\ N &= 3x+2y\end{aligned}}$

In order to be an Exact equation, the partial derivative of *M* with respect to *y* must equal the partial derivative of *N* with respect to *x*:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

To verify that we have an Exact equation, we find:

$$\tfrac{d}{dy}(2x+3y) = 3$$
$$\tfrac{d}{dx}(3x+2y) = 3$$

THE SOLUTION

To solve, we first integrate *M* with respect to *x* and use *g*(*y*) for the constant of integration:

$$\int M\ dx = \int (2x+3y)\,dx = x^2 + 3xy + g(y)$$

Now we take this result and differentiate with respect to *y*:

$$\tfrac{d}{dy}[x^2 + 3xy + g(y)] = 3x + g'(y)$$

We set this equal to *N* and solve for *g*'(*y*):

$$3x + g'(y) = 3x + 2y$$
$$g'(y) = 2y$$

We integrate to find *g*(*y*):

$$g(y) = \int g'(y) = \int 2y\ dy = y^2 + C_1$$

Substitute this into the first expression containing *g*(*y*) to obtain *F*(*x*, *y*):

$$F(x,y) = x^2 + 3xy + g(y) = x^2 + 3xy + y^2 + C_1$$

If an initial condition is given, the value of C_1 can be found, yielding a particular solution. In this case, an initial condition was not given.

THE ANSWER

The general solution is then written in this form, absorbing the value of C_1 into *C*.

$$\boxed{x^2 + 3xy + y^2 = C}$$

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