AC CIRCUITS

ELTE 1402

Frequency and Period:

$$T = \frac{1}{f}$$

where: T = time of one period [s]
f = frequency [*hertz* Hz]

Angular Velocity: [radians/second] $\omega = 2\pi f$

Wavelength: [meters]

$$\lambda = \frac{v}{f}$$

where: λ = wavelength [m]
v = velocity [m/s]
f = frequency [Hz]

The velocity of *radio waves* is 3×10^8 m/s.
The velocity of *sound waves* is 1130 m/s

Average Value of a Sine Wave: [volts]

$$e_{avg} = 0.637E_{\max} = \frac{2}{\pi}E_{\max}$$

Average Current of a Sine Wave: [amps]

$$i_{avg} = 0.637I_{\max} = \frac{2}{\pi}I_{\max}$$

Effective or *rms* Value of a Sine Wave: [volts] The value of alternating current that produces the same heating effect as the corresponding value of direct current.

$$e_{rms} = E_{\max}\sin 45 \quad \text{or} \quad e_{rms} = 0.707E_{\max} = \frac{1}{\sqrt{2}}E_{\max}$$

$E_{\max}$ is peak voltage $E_{\max} \times 2 = E_{pp}$ (peak to peak)

AC voltage values will assumed *rms* unless otherwise specified.

Instantaneous Sinusoidal Voltage: [volts]

$e = E_{\max}\sin\varpi t$ (radian mode) or

$e = E_{\max}\sin\theta$ (degree mode)

where: ω = angular velocity [rad/s] or $2\pi f$
t = time in motion [sec.]
θ = angle [degrees]

The resultant instantaneous voltage of two waveforms can be found by expressing in the above form and adding the two together.

Vector: A straight line representing the magnitude and direction of a quantity.

Phasor: A quantity that has magnitude and direction in the time domain. i.e. a *vector* that changes with time.

DC Component of Rectified AC is the average value of the Voltage.

$$E_{avg} = \frac{E_{\max}}{\pi} \quad \text{half-wave rectifier}$$

$$E_{avg} = \frac{2E_{\max}}{\pi} \quad \text{full-wave rectifier}$$

Reactance

A coil has an inductive reactance in ohms of: $\boxed{X_L = 2\pi fL = \omega L}$ (L = Henrys)

A capacitor has a capacitive reactance in ohms of: $\boxed{X_C = \frac{1}{2\pi fC}}$

The resonant frequency of an inductor and capacitor in series or parallel: $\boxed{f = \frac{1}{2\pi\sqrt{LC}}}$

Power

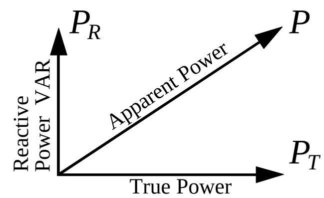
Power in a Resistive Circuit:

$P = ei$ instantaneous power

$P_{avg} = \frac{E_{\max}\,I_{\max}}{2} = E_{rms}\,I_{rms}$ power dissipated

Power Factor is the ratio of true power to apparent power. Multiply by 100 to express as a percent. A power factor of 1 is desirable to keep the size of power transformers and wiring to a minimum. This is done by adding capacitance to an inductive circuit

$$\boxed{= \cos\theta = \frac{R}{Z} = \frac{r}{V} = \frac{P_T}{P_A}}$$



True Power is the product of voltage and current and power factor; the actual power consumed in a purely resistive ac circuit.

$$P_T = VI \times PF$$

$$P_T = \frac{v_R^2}{R} = EI\cos\theta$$

Apparent Power is the product of voltage and current when they are not in phase. $P_A = EI$

Volt-Ampere Reactive (VAR) Power is the the vertical component of product of E and I in a reactive ac circuit: $P_R = EI\sin\theta$

<u>Voltage-Ratio</u> formula, by the $\frac{\quad}{X} = \frac{V_R}{\quad} = I$ throughout a series circuit, where X X or X :

<u>Phase Angle</u>: $\tan\theta = \frac{X}{R} = \frac{V_X}{V_R}$ $\cos\theta = \frac{V_R}{V} = \frac{R}{Z}$

(θ would be negative in a capacitive circuit)

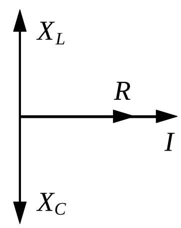
<u>Impedance</u>: $Z^2 = R^2 + X^2$ $E = I\,Z$

$V^2 = V_R{}^2 + V_X{}^2$ $\frac{Z}{V} = \frac{X_C}{V_C} = \frac{R}{V_R}$

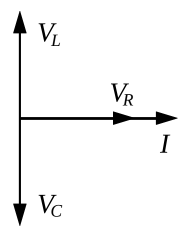
In rectangular notation: $Z = R \pm \mathrm{j}X$

Series RCL Circuits

The <u>Resultant Phasor</u> $X = X_L - X_C$ is in the direction of the larger reactance and determines whether the circuit is inductive or capacitive. If X_L is larger than X_C, then the circuit is inductive and X is a vector in the upward direction.



In series circuits, the amperage is the reference (horizontal) vector. This is observed on the oscilloscope by looking at the voltage across the resistor. The two vector diagrams at right illustrate the phase relationship between voltage, resistance, reactance, and amperage.



<u>Impedance</u>: $Z^2 = R^2 + (X_L - X_C)^2$ $Z = \frac{R}{\cos\theta}$

Impedance may be found by adding the components using vector algebra. By converting the result to polar notation, the <u>phase angle</u> is also found.

For <u>multielement circuits</u>, total each resistance and reactance before using the above formula.

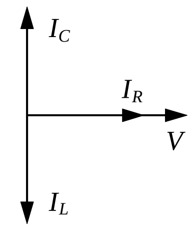
Parallel RC and RL Circuits

$I_T = \sqrt{I_R{}^2 + I_X{}^2}$ $\tan\theta = \frac{I_X}{I_R}$ $I\angle -\theta = \frac{V\angle 0°}{Z\angle\theta}$

Parallel RCL Circuits

$I_T = \sqrt{I_R{}^2 + (I_C - I_L)^2}$

$\tan\theta = \frac{I_C - I_L}{I_R}$



To find total current and phase angle in <u>multielement circuits</u>, find I for each path and add vectorally. Note that when converting between current and resistance, a division will take place requiring the use of polar notation and resulting in a change of sign for the angle since it will be divided into (subtracted from) an angle of zero.

<u>Equivalent Series Circuit</u>: Given the Z in polar notation of a parallel circuit, the resistance and reactance of the equivalent series circuit is as follows:

$R = Z_T \cos q$ $X = Z_T \sin q$

Resonant Circuits

<u>Resonant Frequency</u>: The frequency at which $X_L = X_C$. In a **series**-resonant circuit, the impedance is at its minimum and the current is at its maximum. For a **parallel**-resonant circuit, the opposite is true.

$$\boxed{f_R = \frac{1}{2\pi\sqrt{LC}}}$$

Circuit Q is the **quality of the circuit**, the ratio of inductive reactance of a coil to its resistance. A Q of 10 or greater is considered high Q.

$$\boxed{Q = \frac{X_L}{r_s} = \frac{E_{capacitor}}{E_{applied}}}$$

r_s is the series resistance (applies to a coil or a resonant LC circuit)

If a capacitor is added to form a series-resonant circuit, the Q remains the same.

In a parallel resonant circuit, Q can be expressed as: $Q = \frac{I_L}{I_T} = \frac{I_C}{I_T}$

<u>Line Current</u> in a parallel-resonant circuit: $I_T = I_C - I_L$

<u>Bandwidth</u>

series-resonant circuit: The frequency range between the two points on the frequency response curve that are .707 of the maximum *current*.

$$\boxed{BW = \frac{f_r}{Q}}$$

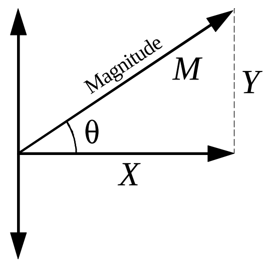
parallel-resonant circuit: The frequency range between the points where the *impedance* is .707 of the maximum.

The resonant frequency f_r is at the center of the bandwidth.

Vector Algebra

Rectangular Notation: $Z = R \pm \mathbf{j}X$ where +j represents inductive reactance and -j represents capacitive reactance. For example, $Z = 8 + \mathbf{j}6\Omega$ means that a resistor of 8Ω is in series with an inductive reactance of 6Ω.

Polar Notation: $Z = M \angle\theta$, where M is the magnitude of the reactance and θ is the direction with respect to the horizontal (pure resistance) axis. For example, a resistor of 4Ω in series with a capacitor with a reactance of 3Ω would be expressed as $5 \angle{-36.9°}\ \Omega$.



In the descriptions above, impedance is used as an example. Rectangular and Polar Notation can also be used to express amperage, voltage, and power.

To convert from **rectangular to polar** notation:

Given: X - $\mathbf{j}Y$ (careful with the sign before the "**j**")

Magnitude: $\sqrt{X^2 + Y^2} = M$

Angle: $\tan\theta = \frac{-Y}{X}$ (negative sign carried over from rectangular notation in this example)

Note: Due to the way the calculator works, if X is negative, you must **add 180°** after taking the inverse tangent. If the result is greater than 180°, you may optionally subtract 360° to obtain the value closest to the reference angle.

To convert from **polar to rectangular** (j) notation:

Given: $M \angle\theta$

X Value: $M \cos\theta$

Y (j) Value: $M \sin\theta$

In conversions, the **j** value will have the same sign as the θ value for angles having a magnitude < 180°.

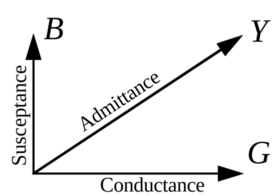
Use rectangular notation when adding and subtracting.

Use polar notation for multiplication and division. Multiply in polar notation by multiplying the magnitudes and adding the angles. Divide in polar notation by dividing the magnitudes and subtracting the denominator angle from the numerator angle.

Conductance (G): The reciprocal of resistance in siemens (S).

Susceptance (B, B_L, B_C): The reciprocal of reactance in siemens (S).

Admittance (Y): The reciprocal of impedance in siemens (S).



Using Thévenin's Theorem

Consider the load to be an open circuit. Under this condition, calculate the voltage present at the load terminals (called the *Thévenin voltage*, E_{TH}).

Consider the load to be an open circuit **and** the AC power source to be a short circuit. Now calculate the impedance seen at the load terminals (called the *Thévenin impedance*, Z_{TH}).

Consider the new circuit below, with E_{TH} as the voltage and Z_{TH} as the impedance of a reactance in series. The voltage and current across R_L may now be calculated using the *Voltage Divider* theorem.

Thévenin Equivalent Circuit