LOGARITHMS AND EXPONENTIAL FUNCTIONS

Definition of Exponential Function: $f(x) = b^x$ where x and b are real numbers and $b > 0$ and $b \neq 1$. The domain is the set of all real numbers and the range is the set of all positive real numbers.

Exponential Function Theorems:

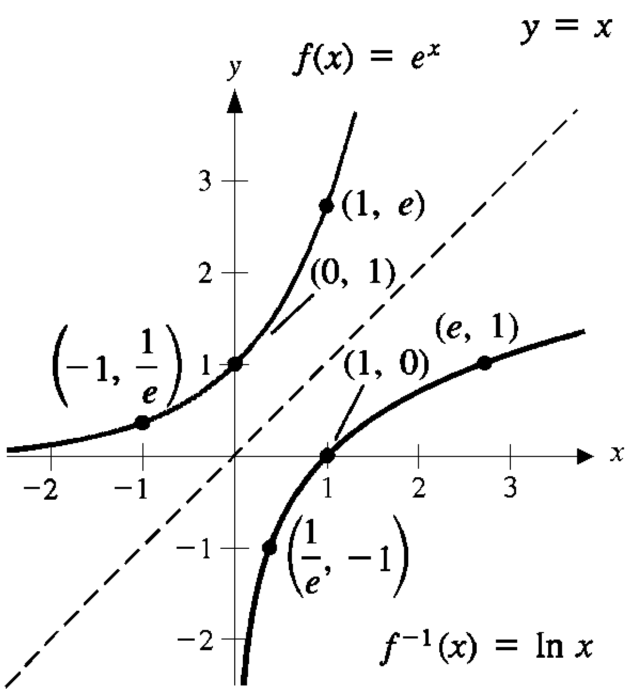
$b^x b^y = b^{x+y}$ $\qquad (b^x)^y = b^{xy}$

$\dfrac{b^x}{b^y} = b^{x-y}$ $\qquad b^x > 0$

$b^x = b^y$ if and only if $x = y$

An Inverse Function $f^{-1}(x)$ is the relation obtained by interchanging the components of the ordered pairs of a one-to-one function.

Definition of a Logarithm Function: The inverse of the exponential function, written $\log_b x = y$, where $x = b^y$; $x > 0$, $b > 0$, $b \neq 1$. The domain is the set of all positive real numbers and the range is the set of all real numbers.



Common Logarithms: The values of $\log_{10} x$ are called common logarithms or logarithms to the base 10. The numeral 10 designating the base is usually omitted: $\log x = \log_{10} x$.

Common Logarithm Theorem: If k is any real number and x is any positive real number, then $\log_x(10^k) = k + \log_x$.

If $\log_x(10^k) = k + \log_x$ where k is an integer and x is a real number such that $1 \leq x < 10$, then k is called the *characteristic* of $\log_x(10^k)$ and $\log_x$ is called the *mantissa* of $\log_x(10^k)$.

Natural Logarithmic Function $\quad f(x) = \log_e x = \ln x$

The **natural number *e*** ≈ 2.71828182846. To get this number on the calculator, press `1 INV lnx`. $\log_e x$ is written $\ln x$ (read "el - en - ex")

$e = \lim_{x \to 0}(1+x)^{1/x}$ $\qquad \ln x = b$ if and only if $e^b = x$

$\lim_{x \to 0^+} \ln x = -\infty$ $\qquad \lim_{x \to \infty} \ln x = \infty$.

$\ln e^x = x$ $\qquad e^{a \ln b} = b^a$

$\ln xy = \ln x + \ln y$ $\qquad \ln \dfrac{x}{y} = \ln x - \ln y$

$\ln x^y = y \ln x$

Logarithms to other bases:

$y = \log_a x$ if and only if $a^y = x$

$\log_a xy = \log_a x + \log_a y$ $\qquad \log_a \dfrac{x}{y} = \log_a x - \log_a y$

$\log_a x^y = y \log_a x$ $\qquad \log_a x = \dfrac{\log_b x}{\log_b a}$

A **calculator** can be used to evaluate an expression such as $\log_2 14$ by virtue of the fact that it is equivalent to $\ln 14 / \ln 2$.

Examples of Logarithms:

$\log_{10} 1000 = 3$

$\log_2\left(\frac{1}{16}\right) = -4$

$\log_{125}\left(\frac{1}{25}\right) = -\frac{2}{3}$

Miscellaneous: $\in$ means *is an element of.*

Reference: *Precalculus Mathematics* CTC Library QA 39.2 G78, pp 184-201

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