

TRIGONOMETRIC IDENTITIES

The six trigonometric functions:

$$\begin{aligned} \sin \theta &= \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} & \csc \theta &= \frac{\text{hyp}}{\text{opp}} = \frac{r}{y} = \frac{1}{\sin \theta} \\ \cos \theta &= \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} & \sec \theta &= \frac{\text{hyp}}{\text{adj}} = \frac{r}{x} = \frac{1}{\cos \theta} \\ \tan \theta &= \frac{\text{opp}}{\text{adj}} = \frac{y}{x} = \frac{\sin \theta}{\cos \theta} & \cot \theta &= \frac{\text{adj}}{\text{opp}} = \frac{x}{y} = \frac{1}{\tan \theta} \end{aligned}$$

Sum or difference of two angles:

$$\begin{aligned} \sin(a \pm b) &= \sin a \cos b \pm \cos a \sin b \\ \cos(a \pm b) &= \cos a \cos b \mp \sin a \sin b \\ \tan(a \pm b) &= \frac{\tan a \pm \tan b}{1 \mp \tan a \tan b} \end{aligned}$$

Double angle formulas:

$$\begin{aligned} \tan 2\theta &= \frac{2 \tan \theta}{1 - \tan^2 \theta} \\ \sin 2\theta &= 2 \sin \theta \cos \theta & \cos 2\theta &= 2 \cos^2 \theta - 1 \\ \cos 2\theta &= 1 - 2 \sin^2 \theta & \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \end{aligned}$$

Pythagorean Identities:

$$\begin{aligned} \tan^2 \theta + 1 &= \sec^2 \theta & \sin^2 \theta + \cos^2 \theta &= 1 \\ \cot^2 \theta + 1 &= \csc^2 \theta \end{aligned}$$

Half angle formulas:

$$\begin{aligned} \sin^2 \theta &= \frac{1}{2}(1 - \cos 2\theta) & \cos^2 \theta &= \frac{1}{2}(1 + \cos 2\theta) \\ \sin \frac{\theta}{2} &= \pm \sqrt{\frac{1 - \cos \theta}{2}} & \cos \frac{\theta}{2} &= \pm \sqrt{\frac{1 + \cos \theta}{2}} \\ \tan \frac{\theta}{2} &= \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}} = \frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin \theta} \end{aligned}$$

Sum and product formulas:

$$\begin{aligned} \sin a \cos b &= \frac{1}{2}[\sin(a+b) + \sin(a-b)] \\ \cos a \sin b &= \frac{1}{2}[\sin(a+b) - \sin(a-b)] \\ \cos a \cos b &= \frac{1}{2}[\cos(a+b) + \cos(a-b)] \\ \sin a \sin b &= \frac{1}{2}[\cos(a-b) - \cos(a+b)] \\ \sin a + \sin b &= 2 \sin\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right) \\ \sin a - \sin b &= 2 \cos\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right) \\ \cos a + \cos b &= 2 \cos\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right) \\ \cos a - \cos b &= -2 \sin\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right) \end{aligned}$$

Law of cosines:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

where A is the angle of a scalene triangle opposite side a.

Radian measure:

$$\begin{aligned} 8.1 \text{ p420} \quad 1^\circ &= \frac{\pi}{180} \text{ radians} \\ 1 \text{ radian} &= \frac{180^\circ}{\pi} \end{aligned}$$

Reduction formulas:

$$\begin{aligned} \sin(-\theta) &= -\sin \theta & \cos(-\theta) &= \cos \theta \\ \sin(\theta) &= -\sin(\theta - \pi) & \cos(\theta) &= -\cos(\theta - \pi) \\ \tan(-\theta) &= -\tan \theta & \tan(\theta) &= \tan(\theta - \pi) \\ \mp \sin x &= \cos\left(x \pm \frac{\pi}{2}\right) & \pm \cos x &= \sin\left(x \pm \frac{\pi}{2}\right) \end{aligned}$$

Complex Numbers:

$$\begin{aligned} e^{\pm j\theta} &= \cos \theta \pm j \sin \theta \\ \cos \theta &= \frac{1}{2}(e^{j\theta} + e^{-j\theta}) & \sin \theta &= \frac{1}{2j}(e^{j\theta} - e^{-j\theta}) \end{aligned}$$

TRIGONOMETRIC VALUES FOR COMMON ANGLES

Degrees	Radians	sin q	cos q	tan q	cot q	sec q	csc q
0°	0	0	1	0	Undefined	1	Undefined
30°	π/6	1/2	√3/2	√3/3	√3	2√3/3	2
45°	π/4	√2/2	√2/2	1	1	√2	√2
60°	π/3	√3/2	1/2	√3	√3/3	2	2√3/3
90°	π/2	1	0	Undefined	0	Undefined	1
120°	2π/3	√3/2	-1/2	-√3	-√3/3	-2	2√3/3
135°	3π/4	√2/2	-√2/2	-1	-1	-√2	√2
150°	5π/6	1/2	-√3/2	-√3/3	-√3	-2√3/3	2
180°	π	0	-1	0	Undefined	-1	Undefined
210°	7π/6	-1/2	-√3/2	√3/3	√3	-2√3/3	-2
225°	5π/4	-√2/2	-√2/2	1	1	-√2	-√2
240°	4π/3	-√3/2	-1/2	√3	√3/3	-2	-2√3/3
270°	3π/2	-1	0	Undefined	0	Undefined	-1
300°	5π/3	-√3/2	1/2	-√3	-√3	2	-2√3/3
315°	7π/4	-√2/2	√2/2	-1	-1	√2	-√2
330°	11π/6	-1/2	√3/2	-√3/3	-√3	2√3/3	-2
360°	2π	0	1	0	Undefined	1	Undefined

Expansions for sine, cosine, tangent, cotangent:

$$\sin y = y - \frac{y^3}{6} + \frac{y^5}{5!} - \frac{y^7}{7!} + \dots$$

$$\cos y = 1 - \frac{y^2}{2} + \frac{y^4}{4!} - \frac{y^6}{6!} + \dots$$

$$\tan y = y + \frac{y^3}{3} + \frac{2y^5}{15} + \dots$$

$$\cot y = \frac{1}{y} - \frac{y}{3} + \frac{y^3}{45} - \frac{2y^5}{945} - \dots$$

Hyperbolic functions:

$$\sinh y = \frac{1}{2}(e^y - e^{-y}) \quad \sinh jy = j \sin y$$

$$\cosh y = \frac{1}{2}(e^y + e^{-y}) \quad \cosh jy = j \cos y$$

$$\tanh jy = j \tan y$$

Expansions for hyperbolic functions:

$$\sinh y = y + \frac{y^3}{6} + \dots$$

$$\cosh y = 1 + \frac{y^2}{2} + \dots$$

$$\operatorname{sech} y = 1 - \frac{y^2}{2} + \frac{5y^4}{24} - \dots$$

$$\operatorname{ctnh} y = \frac{1}{y} + \frac{y}{3} - \frac{y^3}{45} + \dots$$

$$\operatorname{csch} y = \frac{1}{y} - \frac{y}{6} + \frac{7y^3}{360} - \dots$$

$$\tanh y = y - \frac{y^3}{3} + \frac{2y^5}{15} - \dots$$