Smith Chart

Using the Smith Chart

First **normalize the load** impedance by dividing by the characteristic impedance, and find this point on the chart.

When working in terms of **reactance**, an **inductive load** will be located on the top half of the chart, a **capacitive load** on the bottom half. It's the other way around when working in terms of **susceptance**.

Draw a **straight line** from the center of the chart through the normalized load impedance point to the edge of the chart.

Anchor a compass at the center of the chart and **draw a circle** passing through the normalized load impedance point. Points along this circle represent the normalized impedance at various points along the transmission line. **Clockwise movement** along the circle represents movement from the load toward the source, with one full revolution representing **1/2 wavelength** as marked on the outer edge of the Smith chart. The two points where the circle intersects the horizontal axis are the **voltage maxima** (right) and the **voltage minima** (left).

The point opposite the **impedance** (180° around the arc) is the **admittance**. The reason admittance (or susceptance) is useful is because admittances in parallel are simply added. (Admittance is the reciprocal of impedance; susceptance is the reciprocal of reactance.)

Reflection coefficient:

$$\rho = \frac{Z - Z_0}{Z + Z_0} = \underbrace{\frac{r + j \cdot x - 1}{r + j \cdot x + 1}}_{\text{normalized form}}$$

$$\underbrace{r}_{\text{real}} + \underbrace{j \cdot x}_{\text{imag.}} = \underbrace{\frac{1 - \left(u^2 + v^2\right)}{\left(1-u\right)^2 + v^2}}_{\text{real}} + \underbrace{j \cdot \frac{2 \cdot v}{\left(1-u\right)^2 + v^2}}_{\text{imaginary}}$$

Reflection coefficient in the complex plane:

$$\rho = u + j \cdot v$$

Normalized load impedance:

$$R_{L(norm)} = \frac{1 + \rho}{1 + \rho} = r + j \cdot x$$

Input impedance:

$$Z_{in} = Z_0 \frac{1 + \rho}{1 - \rho} = Z_0 \frac{Z_L + jZ_0 \tan\left(\beta l\right)}{Z_0 + jZ_L \tan\left(\beta l\right)}$$

$$\text{where } \beta\lambda = 2\pi$$

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The *u* and *v* axes are normally not shown on the Smith Chart. With respect to these axes, ρ is zero when at the origin and one when on the outer circle. The distance from ρ to the origin in the magnitude of ρ. If ρ is greater than one, that means there is gain.

Using the Smith chart, we assume α=0, otherwise the circular path taken around the chart would be a spiral. α is the real part of the **complex propagation constant** γ where $\gamma = \sqrt{ZY} = \alpha + j\beta$ and $\beta\lambda = 2\pi$.