# ELECTRONIC CIRCUITS II EE338K 

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| FEEDBACK TOPOLOGIES |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| AMPLIFIER TYPE | INPUT | FED BACK AS | OUTPUT | MONITORS | $\mathbf{Z}_{\text {IN }}$ | $\mathbf{Z}_{\text {OUT }}$ |
| VCVS Voltage controlled voltage source, inverting | parallel | current | parallel | voltage | 0 | 0 |
| VCVS Voltage controlled voltage source, non-invert. | series | voltage | parallel | voltage | $\infty$ | 0 |
| CCVS Current controlled voltage source | parallel | current | parallel | voltage | 0 | 0 |
| VCCS Voltage controlled current source | series | voltage | series | current | $\infty$ | $\infty$ |
| CCCS Current controlled current source | parallel | current | series | current | 0 | $\infty$ |

Parallel input - input signal always goes into the negative input
Series input - input signal always goes into the positive input

## MORE TERMINOLOGY

Gain margin is the number of decibels the loop gain magnitude is below 0 dB at the frequency of phase reversal. 10 dB is considered safe.
Phase margin is the number of degrees the loop gain phase change is short of phase reversal at the frequency at which the magnitude of the loop gain is unity. $45^{\circ}$ or more is considered safe.

## AMOUNT OF FEEDBACK

$$
\begin{aligned}
& \text { feedback }(\mathrm{dB})=20 \log _{10} \frac{A}{A_{f}} \\
& \quad=20 \log _{10}|A|-20 \log _{10}\left|A_{f}\right|
\end{aligned}
$$

= open loop gain [dB] - closed loop gain [dB]

## FEEDBACK TOPOLOGIES

Four of the amplifier types described in more detail on the following page provide us with a complete set of the possible arrangements or topologies obtainable in a feedback system.


## OP AMP FEEDBACK CIRCUITS

$A_{f \infty}$ denotes asymptotic gain and means voltage across the inputs and current into the inputs are zero, $A \rightarrow \infty$.
$A_{f}$ denotes closed loop gain, called finite $A$, and takes into account the small voltage across the inputs $V_{\text {out }}=A\left(V_{+}-V_{-}\right)$
$A$ denotes open loop gain and is the op amp gain, typically about $10^{5}$.
VOLTAGE FOLLOWER OR UNITY GAIN AMPLIFIER
$V_{\text {out }}=A\left(V_{+}-V_{-}\right)$
$\frac{V_{\text {out }}}{A}=V_{\text {ref }}-V_{\text {out }}$


If $A$ is very large then $V_{\text {out }}=V_{\text {ref }}$
VCVS - NON-INVERTING VOLTAGE AMPLIFIER
provides an output voltage proportional to the input voltage.
Series, hi-Z input; parallel, lo-Z out.


We approach asymptotic conditions if $A \gg \frac{R_{1}+R_{f}}{R_{1}}$
VCVS - INVERTING VOLTAGE AMPLIFIER Provides an output voltage that is proportional but of opposite polarity to the input voltage. Parallel, lo-Z input; parallel, lo-Z out.
$\beta=\frac{R_{1}}{R_{1}+R_{f}}$

$A_{f \infty}=\frac{v_{\text {out }}}{v_{\text {in }}}=-\frac{R_{f}}{R_{1}}$

$$
A_{f}=\frac{v_{\text {out }}}{v_{\text {in }}}=\frac{-\frac{R_{f}}{R_{1}+R_{f}}}{\frac{1}{A}+\frac{R_{1}}{R_{1}+R_{f}}}
$$

CCVS - TRANSRESISTANCE AMPLIFIER Current-
controlled voltage source. The output voltage is proportional to the input current. Parallel, lo-Z input; parallel, lo-Z out.
$i_{i n}=i_{f}$
$v_{\text {out }}=-i_{f} R_{f}=-i_{i n} R_{f}$
$A_{f \infty}=\frac{v_{\text {out }}}{i_{\text {in }}}=-R_{f} \mathrm{~V} / \mathrm{A}$
$z_{\text {in }}=R_{\text {in }} \|\left(\frac{R_{f}}{1+A}\right)$

$z_{\text {in }}$ is typically less than $1 \Omega$
VCCS - TRANSCONDUCTANCE AMPLIFIER Voltage-
controlled current source. The output current is proportional to the input voltage. Series, hi-Z input; series, lo-Z out.

$$
\begin{aligned}
& v_{\text {in }}=v_{+}=v_{-}=i_{L} R_{1} \\
& i_{L}=\frac{v_{\text {in }}}{R_{1}} \\
& A_{f \infty}=\frac{i_{L}}{v_{\text {in }}}=\frac{1}{R_{1}} \mathrm{~A} / \mathrm{v} \stackrel{v_{i n}}{=} A_{R_{1}}=\frac{1}{R_{1}+R_{L}} \times \frac{1}{\frac{1}{A}+\frac{R_{1}}{R_{1}+R_{L}}}
\end{aligned}
$$

We approach asymptotic conditions if $A \gg \frac{R_{1}+R_{L}}{R_{1}}$
CCCS - CURRENT AMPLIFIER Provides a current output proportional to input current. Parallel, lo-Z input; series hi-Z output.
$i_{i n}=i_{f}$
$v_{1}=-i_{f} R_{f}=-i_{i n} R_{f}$
$v_{1}=\left(i_{f}+i_{L}\right) R_{1}$
$=\left(i_{i n}+i_{L}\right) R_{1}$

$A_{f \infty}=\frac{i_{L}}{i_{\text {in }}}=-\frac{R_{1}+R_{f}}{R_{1}} A_{f}=-\frac{\frac{R_{1}+R_{f}}{R_{1}}+\frac{1}{A}}{1+\frac{1}{A}\left(1+\frac{R_{L}}{R_{1}}\right)} \mathrm{A} / \mathrm{A}$
We approach asymptotic conditions if $A \gg 1+\frac{R_{L}}{R_{1}}$

## GAIN ERROR

$$
E \% \equiv \frac{\left(A_{f \infty}-A_{f}\right)}{A_{f \infty}} \times 100=\frac{100}{1-L}
$$

## A BOX - $\beta$ BOX MODELING

The A box models the op amp and the b box models the feedback circuit as an ideal amplifier. The system shown models a non-inverting voltage-controlled voltage source topology (series input, parallel output), which is the most commonly used. This configuration is the basis for much discussion to follow.


## D DESENSITIVITY FACTOR

feedback has the effect of desensitizing the gain of the system to changes in the A box gain (see FEEDBACK AND GAIN). Input and output impedance are also affected by a factor of $D$ with the application of feedback. The desensitivity factor appears in many formulas as $1+\beta A$

$$
\begin{array}{rlrl}
D & =1+\beta A & & A=\text { open loop gain } \\
& =1-L & & \beta=\text { b box gain } \\
& & L=\text { loop gain (a negative value) }
\end{array}
$$

## $\beta$ BETA and L LOOP GAIN

When the feedback circuit is thought of as a separate amplifier as shown in the A-box/ $\beta$-box schematic, $\beta$ is the gain of the feedback circuit. The value of $\beta$ is normally less than one, and the small feedback signal is combined with the input signal $180^{\circ}$ out of phase, which effectively reduces the gain of the amplifier (among other consequesces). $L$ is called the loop gain and represents the amplifier gain once through the A-box/ $\beta$-box loop. The value of $L$ is always a negative number.

$$
\begin{aligned}
L=-\beta A & & A=\text { open loop gain or A box gain } \\
=1-D & & \beta=\text { b box gain } \\
& & =\text { loop gain (a negative value) } \\
& & D=\text { desensitivity factor }
\end{aligned}
$$

## FEEDBACK AND GAIN

The formula for closed loop gain of the $A$ box $-\beta$ box shown previously can be rewritten as follows:

$$
A_{f}=\frac{v_{\text {out }}}{v_{\text {in }}}=\frac{1}{\frac{1}{A}+\beta}
$$

With the value of $A$ very large and the value of $\beta$ somewhat less than one, the term $1 / A$ will be much smaller than $\beta$. Therefore, with the use of feedback, the value of the gain $A$ of the op amp (A box) now has little impact on the overall gain of the circuit.

## FEEDBACK AND FREQUENCY RESPONSE

As shown below, feedback has the effect of extending the bandwidth by a factor of $1+\beta A$. However, since the gain is reduced by a factor of $1+\beta A$, the net result is that the gain bandwidth product remains constant, feedback or not.

$$
A_{f}(j \omega)=\frac{1}{\frac{1}{A(j \omega)}+\beta}
$$

Transfer function for an
A box with one high frequency corner:

$$
A(j \omega)=\frac{A_{0}}{1+j \frac{f}{f_{H}}}
$$

Adding feedback has the following effect:
$A_{0 f}(j \omega)=\frac{A_{0}}{1+\beta A_{0}}$
$f_{H f}=\left(1+\beta A_{0}\right) f_{H}$
$f_{L f}=\frac{f_{L}}{1+\beta A_{0}}$
$A_{f}(\mathrm{j} \omega)=$ closed loop gain (sinusoidal steady state response)
$A=$ open loop or A-box gain
$A_{0}=$ open loop passband gain
$A_{0 f}=$ closed loop gain in the passband
$\beta=b$ box gain
$f=$ frequency [Hz]
$f_{H}=$ high corner frequency, the frequency at which the gain has dropped 3 dB below the passband gain without feedback [Hz]
$f_{H f}=$ high corner frequency, with feedback [Hz]
$f_{L}=$ low corner frequency $[\mathrm{Hz}]$
$f_{L f}=$ low corner frequency, with feedback [Hz]

## FEEDBACK WITH MULTIPLE CORNER FREQUENCIES

Transfer function for an A
box with $n$ high frequency corners occurring at the same frequency:

$$
A(j \omega)=\frac{A_{0}}{\left(1+j \frac{f}{f_{H}}\right)^{n}}
$$

with 1 low freq. corner:

$$
A(j \omega)=\frac{A_{0}}{\left(1-j \frac{f_{L}}{f}\right)}
$$

Finding the frequency of phase reversal $f_{0}$ for the first function above:
$-180^{\circ}=-n \tan ^{-1}\left(\frac{f_{0}}{f_{H}}\right)$
$A_{f}(\mathrm{j} \omega)=$ closed loop gain (sinusoidal steady state response)
$A=$ open loop gain
$A_{0}=$ open loop gain in the passband
$A_{0 f}=$ closed loop gain in the passband
$\beta=\beta$-box gain
$f=$ frequency [Hz]
$f_{0}=$ frequency of phase reversal [Hz]
$f_{H}=$ high corner frequency, the frequency at which the gain has dropped 3 dB below the passband gain [Hz]
$f_{L}=$ low corner frequency [Hz]
$n=$ number of high frequency corners at a single frequency $f_{0}$ [Hz]

Transfer function for an A box with 3 high frequency corners occurring at different frequencies:
$A(j \omega)=\frac{A_{0}}{\left(1+j \frac{f}{f_{H 1}}\right)\left(1+j \frac{f}{f_{H 2}}\right)\left(1+j \frac{f}{f_{H 3}}\right)}$

## IMPEDANCE

## FEEDBACK AND INPUT IMPEDANCE

This applies to the A box $-\beta$ box voltage amplifier circuit shown previously. Note that the addition of feedback in the circuit results in an increase in input impedance by a factor of $(1+\beta A)$.

$$
\begin{array}{cl}
Z_{\text {in } f}=\frac{V_{\text {in }}}{I_{\text {in }}} & \begin{array}{l}
Z_{\text {inf }}=\text { input impedance with } \\
\text { feedback }[\Omega]
\end{array} \\
=Z_{\text {in } A}(1+\beta A) & \begin{array}{r}
Z_{\text {inA }}=\text { input impedance without } \\
\text { feedback }[\Omega] \\
\\
\hline
\end{array} \\
& \begin{array}{l}
\text { open loop gain }
\end{array} \\
\hline \text {-box gain }
\end{array}
$$

## FEEDBACK AND OUTPUT IMPEDANCE

This applies to the A box - $\beta$ box voltage amplifier circuit shown previously. Note that the addition of feedback in the circuit results in a reduction of output impedance by a factor of $(1+\beta A)$.

$$
\begin{array}{cl}
Z_{\text {out } f}=\frac{V_{\text {out o/c }}}{I_{\text {out } s / c}} & \begin{array}{l}
Z_{\text {out } f}=\text { output impedance with feedback } \\
{[\Omega]}
\end{array} \\
=\frac{Z_{\text {out } A}=\text { output impedance without }}{} \quad \begin{array}{l}
\text { feedback }[\Omega]
\end{array} \\
1+\beta A & \begin{array}{l}
V_{\text {out } o / c}=\text { open circuit output voltage }[\mathrm{V}] \\
I_{\text {out } s / c}=\text { short-circuit output current }[\mathrm{A}] \\
A=\text { open loop gain } \\
\beta=\beta \text {-box gain }
\end{array}
\end{array}
$$

## $L$ FINDING LOOP GAIN

1) Model the op amp like this:

2) If the circuit is parallel input (signal enters negative op amp input) convert the input source to a Norton equivalent. If this circuit is series input, use a Thèvenin equivalent source.
3) Redraw the circuit using 1) and 2).
4) Separate the circuit by breaking the feedback loop, usually on the output side of the feedback resistor.
5) Find the Thèvenin equivalent of the output half of the circuit from the perspective of the point at which the circuit was broken in 4). Replace ( $v_{+}-v_{-}$) with $v_{t}$.
6) Turn off the input source and simplify the input half of the circuit, preserving the nodes $v_{+}$and $v_{-}$, corresponding to the op amp inputs, and label the voltage between them $v_{d}$.
7) Redraw the circuit using 5) and 6), reconnecting the two halves.
8) Using voltage division, find an equivalent expression for $v_{d}$. Solve this expression for $v_{d} / v_{t}$. This is the loop gain $L$ and should be negative.

IMPEDANCE with $Z_{I S O}, Z_{I P O}, Z_{O S O}$, and $Z_{\text {OPO }}$
$Z_{i s o}$ for series input amplifiers (the input signal is going into the positive input), the impedance with the input source removed as seen looking into $\mathrm{R}_{\mathrm{gen}}$, and takes both $R_{\text {gen }}$ and $R_{0}$ into account.

$$
Z_{i n}=(1-L) Z_{i s o}-R_{g e n}
$$

for parallel input amplifiers (the input signal is going into the negative input), the impedance with the sources off and takes both $\mathrm{R}_{\mathrm{gen}}$ and $\mathrm{R}_{\mathrm{o}}$ into account.

$$
Z_{i n}=\frac{1}{\frac{1-L}{Z_{i p o}}-\frac{1}{R_{g e n}}}
$$

for series output amplifiers (amplifier is acting as a current source, $R_{L}$ does not connect to ground), the impedance with sources off as seen looking out of the op amp through $\mathrm{R}_{\mathrm{o}}$, and $\mathrm{R}_{\mathrm{L}}$ and also taking $\mathrm{R}_{\mathrm{gen}}$ into account.

$$
Z_{\text {out }}=(1-L) Z_{\text {oso }}-R_{L}
$$

$Z_{\text {opo }}$ for parallel output amplifiers ( $R_{L}$ connects to ground), the impedance with sources off as seen looking through into the output terminals with $\mathrm{R}_{\mathrm{L}}$ in place and taking $R_{o}$ and $R_{\text {gen }}$ into account.

$$
Z_{\text {out }}=\frac{1}{\frac{1-L}{Z_{\text {opo }}}-\frac{1}{R_{L}}}
$$

The use of $Z_{i s o}, Z_{i p o}, Z_{o s o}$, and $Z_{o p o}$ permit impedance calculations for amplifiers whose gain is affected by the input and output loads. Note that the word "sources" used here refers to the input source and to voltage source $A$ inside the op amp. Refer to previous section for finding loop gain $L$.

## FREQUENCY

## OP AMP FREQUENCY RESPONSE

There are no coupling capacitors or bypass capacitors in an op amp, so there are no low frequency corners. That is, the passband gain extends all the way to DC.
There are several high frequency corners, but one of them (due to Cc ) is very much lower than the other corners
$f_{u}$ is the unity gain crossover frequency, see page 9 .


## FREQUENCY COMPENSATION to achieve AMPLIFIER STABILITY

An amplifier is unstable if $180^{\circ}$ phase reversal occurs at a frequency where the gain is $1 \mathrm{~V} / \mathrm{V}$ or greater. Viewed on a Nyquist plot, this situation occurs when the plot encircles the point ( $-1,0$ ). Oscillation will occur. There are several ways to stabilize the amplifier:

1. Gain reduction - Reduce the overall gain to obtain a gain margin of 10 dB at the frequency of phase reversal.
2. Phase Lag compensation - Keep all the gain magnitude and sacrifice bandwidth. This is done by adding an additional high frequency corner-at a very low frequency. This method is used when none of the existing corners can be lowered, as in an op amp where circuitry is internal.
Original Loop Gain Transfer Function:
$-L(j \omega)=\frac{K}{\left(1+j \frac{f}{10^{6}}\right)^{3}}$
Phase-Lag compensated Loop Gain Transfer Function:
$-L(j \omega)=\frac{K}{\left(1+j \frac{f}{10}\right)\left(1+j \frac{f}{10^{6}}\right)^{3}}$
Added frequency
corner
3. Lead-lag method - Reduce the frequency of the lowest frequency corner. This raises the bandwidth by an amount equal to the difference between the two lowest corners (previous to compensation). This can be the most effective method, but is useless if the two corner frequencies are identical.



## BJTs

$$
\begin{gathered}
\boldsymbol{h} \text { PARAMETERS } \\
h_{f e}=\beta=\frac{I_{C}}{I_{B}}=g_{m} r_{\pi}=\frac{I_{C}}{V_{T}} r_{\pi} \quad h_{r e}=\frac{r_{\pi}}{r_{\pi}+r_{\mu}} \\
h_{i e}=r_{\pi}=\frac{1}{g_{i}}=\frac{V_{T}}{I_{B}} \quad \frac{1}{h_{o e}}=r_{0}=\frac{V_{A}}{I_{C}}=\frac{1}{g}
\end{gathered}
$$

## BJT MODELS

HYBRID П
$\boldsymbol{r}_{\mu}$ is often taken to be infinite


## $g_{f}$ FORWARD TRANSCONDUCTANCE

The forward transconductance varies with the offset voltage and is at a maximum ( $g_{f \max }$ ) when the offset voltage is zero.

| $g_{f} \equiv \frac{\partial I_{C}}{\partial V_{B E}}$ | $g_{f}=$ forward transconductance $[\mathrm{A} / \mathrm{V}]$ <br> $I_{C}=$ collector current $[\mathrm{A}$ or mA$]$ |
| :---: | :--- |
| $g_{f}=g_{m}=\frac{I_{C}}{V_{T}}$ | $V_{B E}=$ differential input voltage $[\mathrm{V}]$ <br> $V_{T}=$ thermal emitter voltage <br> mV <br> mV |

## $g_{i}$ INPUT TRANSCONDUCTANCE

The conductance looking into the input.

$$
g_{i} \equiv \frac{\partial I_{B}}{\partial V_{B E}} \quad \begin{aligned}
& g_{i}=\text { input transconductance }[\mathrm{A} / \mathrm{V}] \\
& h_{i e}=\text { ratio of thermal voltage to base } \\
& \text { current }[\Omega]
\end{aligned}
$$

WILSON CURRENT MIRROR

The Wilson Current Mirror substantially improves current regulation and thereby improves the CMRR of the differential amplifier.


## BJT INPUT STAGES

DIFFERENTIAL BJT INPUT STAGE

$-V_{E E} \downarrow I_{Q}$
\#1 IMPROVED BJT INPUT STAGE
Collector resistors are replaced with a current mirror in this differential BJT input stage.

$$
A_{1}=\frac{2 g_{f \max }}{g_{2}+g_{4}+g_{i \text { nnd sage }}}=\frac{2 \times \frac{\alpha I_{Q}}{4 V_{T}}}{\frac{\alpha I_{Q}}{2 V_{A}}+\frac{\alpha I_{Q}}{2 V_{A}}+g_{i \text { ind stage }}}
$$


\#2 IMPROVED BJT INPUT STAGE
Differential BJT input stage with active loads, shown with darlington second stage. The addition of $T_{5}$ in the current mirror reduces the base current to $T_{6}$ by approximately a factor of $\beta$. DC currents are shown, and are approximate.

$$
A_{1}=\frac{2 g_{f \max }}{g_{2}+g_{4}+g_{\text {idarl }}}=\frac{2 \times \frac{\alpha I_{Q}}{4 V_{T}}}{\frac{\alpha I_{Q}}{2 V_{A}}+\frac{\alpha I_{Q}}{2 V_{A}}+\frac{\alpha I_{Q}}{\beta_{6}\left(\beta_{7}+1\right) 2 V_{T}}}
$$



## DIFFERENTIAL INPUT GAIN

$A_{1}=\frac{2 g_{f \max }}{g_{2}+g_{4}+g_{\text {idarl }}}=\frac{2 \times \frac{I_{C 2}}{2 V_{T}}}{\frac{I_{C 2}}{V_{A}}+\frac{I_{C 4}}{V_{A}}+\frac{I_{B \text { 1st darlington }}}{2 V_{T}}}$

## $Z_{\text {in diff }}$ DIFFERENTIAL INPUT IMPEDANCE

$$
Z_{\text {indiff }}=2 h_{i e}=\frac{4(\beta+1) V_{T}}{I_{Q}}
$$

## $g_{f}, g_{f \max }$ FORWARD TRANSCONDUCTANCE of a DIFFERENTIAL INPUT AMPLIFIER

The differential input amplifier can be thought of as a transconductance amplifier (VCCS).

$$
\begin{array}{cl}
g_{f}=\left|\frac{\partial I_{C}}{\partial V_{i}}\right| & \begin{array}{l}
g_{f}=\text { forward transconductance }[\mathrm{A} / \mathrm{V} \text { or } \\
\mathrm{mA} / \mathrm{V}] \\
I_{C}=\text { collector current }[\mathrm{A} \text { or } \mathrm{mA}] \\
V_{i}=\text { differential input voltage }[\mathrm{V}]
\end{array} \\
g_{f}=\frac{I_{C 1} I_{C 2}}{\alpha I_{Q} V_{T}} & \begin{array}{l}
V_{T}=\text { thermal voltage, typically } 26 \mathrm{mV} \\
\alpha=\beta /(\beta+1) \text { ratio of collector current to } \\
\text { emiter current. }
\end{array} \\
g_{f \max }=\frac{\alpha I_{Q}}{4 V_{T}} & \begin{array}{l}
I_{Q}=\text { differential current source }[\mathrm{A} \text { or } \\
\mathrm{mA}]
\end{array}
\end{array}
$$

## $g_{c m}$ COMMON MODE TRANSCONDUCTANCE

 of a DIFFERENTIAL INPUT AMPLIFIERThe differential input amplifier can be thought of as a transconductance amplifier (VCCS).
$g_{c m}=\frac{\Delta I_{C}}{\Delta V_{C M}}$
$g_{c m}=$ common mode
transconductance [A/V]
$I_{C}=$ collector current $[\mathrm{A}]$
$V_{C M}=$ common mode input voltage [V]
$\alpha=\beta /(\beta+1)$ ratio of collector current to emitter current.
$g_{o}=$ current source conductance $[\mathrm{A} / \mathrm{V}]$

## CMRR COMMON MODE REJECTION RATIO

The ratio of differential gain to common mode gain.

$$
C M R R=\frac{\left|A_{d}\right|}{\left|A_{c m}\right|}=\frac{g_{f}}{g_{c m}}
$$

in dB: $C M R R=20 \log \frac{\left|A_{d}\right|}{\left|A_{c m}\right|}$
$A_{d}=$ differential gain
$A_{c m}=$ common mode gain
$g_{f}=$ forward transcon-
ductance [A/V]
$g_{c m}=$ common mode trans-conductance [A/V]

## BJT $2^{\text {ND }}$ STAGES

$2^{\text {nd }}$ STAGE AND OUTPUT STAGE


## $2^{\text {nd }}$ STAGE GAIN (DARLINGTON PAIR)

Referring to the circuit diagram above, the second stage consists of $T_{6}$ and $T_{7}$ in a darlington configuration which is associated with the $g_{f d a r l}$ term. The output also uses darlington pairs for the positiveand negative-going signals. This is the reason for squaring the $(\beta+1)$ term.

$$
\begin{aligned}
& A_{2}=-g_{f \text { darl }} R_{\text {CE load }}=-g_{f 6,7}\left\{\left[(\beta+1)^{2} R_{L}\right] \| \frac{1}{g_{7}}\right\} \\
& A_{2}=-\overbrace{\frac{\overbrace{I_{Q}}}{2 V_{T}}}^{g_{f} \text { dent }} \underbrace{}_{\text {Resistive load placed on 2nd }}\{\left[(\beta+1)^{2} R_{L}\right] \| \frac{\overbrace{V_{A}}^{\alpha I_{Q}}}{1 / g_{7}}\} \\
& \text { stage by output stage. }
\end{aligned}
$$

see Darlington Conductances on page 7.

## $2^{\text {nd }}$ STAGE GAIN (SINGLE TRANSISTOR)

 as in the LM 351 on p 11.$$
\begin{aligned}
A_{2} & =-g_{f 5} R_{\text {CE load }}=-\frac{I_{C 5}}{\left|V_{P}\right|}\left\{\left[\left(\beta_{7}+1\right) R_{L}\right] \| \frac{1}{g_{5}}\right\} \\
& =-\frac{\beta_{5} I_{Q}}{\left(\beta_{5}+2\right)\left|V_{P}\right|}\left\{\left[\left(\beta_{7}+1\right) R_{L}\right] \| \frac{V_{A B J T}}{\beta I_{Q} /(\beta+2)}\right\}
\end{aligned}
$$

## $C_{m i}$ MILLER EFFECT CAPACITANCE $f_{H}$ DOMINANT FREQUENCY CORNER

The Miller Effect capacitance in stage two of the threestage op amp is responsible for the dominant (lowest) high frequency corner of the op amp. Other corners are present but are too high to be of concern.

$$
\begin{array}{cc}
\begin{array}{c}
C_{m i}=C_{C}(1-K)
\end{array} & \begin{array}{l}
C_{m i}=\text { Miller capacitance }[\mathrm{F}] \\
C_{C}=\text { Miller effect } \\
\text { capacitance [F] }
\end{array} \\
f_{H}=\frac{1}{2 \pi C_{m i} R_{m i}} & \begin{array}{c}
f_{H}=\begin{array}{c}
\text { the dominant (lowest) } \\
\text { high frequency corner } \\
\text { [Hz] }
\end{array} \\
A_{2}=\begin{array}{c}
\text { gain of the second } \\
\text { stage }
\end{array} \\
R_{m i}=\text { Miller resistance }[\Omega]
\end{array} \\
R_{m i}=\frac{1}{g_{2}+g_{4}+g_{i d a r l}} &
\end{array}
$$

## $f_{u}$ UNITY GAIN CROSSOVER FREQUENCY or GAIN BANDWIDTH PRODUCT

The unity gain crossover frequency is the (high) frequency at which the gain has fallen to one.

$$
f_{u}=\frac{g_{f \max }}{\pi C c}=A_{0} f_{H}
$$

$g_{f \text { max }}=$ maximum forward transconductance [A/V]
$C_{C}=$ Miller effect capacitance [F]
for small signals (don't use):

$$
f_{u}=\frac{I_{Q}}{4 \pi V_{T} C c}=\frac{S R \alpha}{4 \pi V_{T}}
$$

Gain can be expressed as a function of $f_{u}$. Yes, it is $90^{\circ}$ out of phase due to $f_{H}$ being at around 5 Hz .

$$
\frac{V_{o u t}}{V_{i n}}=j \frac{f_{u}}{f}
$$

$$
\begin{aligned}
& A_{0}= \text { open loop passband } \\
& \text { gain } \\
& f_{H}= \text { the dominant (lowest) } \\
& \text { high corner frequency } \\
& \text { (in this case it is } \\
& \text { associated with } \\
& \text { capacitance } C_{C} \text { ) [Hz] } \\
& V_{T}= \text { thermal voltage, } \\
& \text { typically } 26 \mathrm{mV} \\
& S R= \text { slew rate [V/s] }
\end{aligned}
$$

## $I_{S}$ SCALE CURRENT

The scale current or saturation current is proportional to the junction area and affects the value of $V_{B E}$. Scale current is strongly affected by temperature, as a rule of thumb doubling for every $5^{\circ} \mathrm{C}$ rise in temperature. BJT saturation occurs when the transistor is unable to provide a collector current of $\beta I_{B}$, either because the base current is too high or the voltage supply is too low (my definition). The collector to emitter voltage in saturation is typically about 0.2 V .
$I_{E} \simeq I_{S} e^{V_{B E} / V_{T}}$
$I_{E}=$ emitter current [A or mA]
$I_{S}=$ scale current or saturation current
[ A or mA ]
$I_{E}=I_{S}\left(e^{V_{E E} V_{T}}-1\right)$
$\ln \frac{I_{E}}{I_{S}}=\frac{V_{B E}}{V_{T}}$
$V_{B E}=$ base to emitter voltage [ V ]
$V_{T}=$ thermal voltage, typically 26 mV at room temperature

## SR SLEW RATE

The slew rate is the maximum rate of change of the output voltage, or how fast the output voltage changes in response to a large step at the input.
$S R=\frac{I_{Q}}{C_{C}}$
$S R=$ output voltage rate of change [V/s]
$I_{Q}=$ bias current [A]
$C_{C}=$ Miller effect capacitance [F]

## FPBW FULL POWER BANDWIDTH

The full power bandwidth is the highest frequency at which a rail to rail output is available without distortion.
$S R=$ slew rate; output voltage rate of change [V/s]

$$
F P B W=\frac{S R}{2 \pi V_{\text {RAIL }}}
$$

$V_{R A I L}=$ the maximum voltage to which the output may be driven, usually about 2 volts less than the supply voltage [ $V$ ]

## $f_{\max }$ MAXIMUM FREQUENCY

The maximum frequency without slew rate distortion for a sinewave output signal of $V_{p}$ volts. Slew rate distortion occurs when the rate of change of the output voltage attempts to exceed the slew rate (the maximum rate of the change of the output voltage). The maximum rate of change of a sine wave occurs when the voltage crosses zero. So for a sine wave of frequency $f$ and peak amplitude $V_{p},\left(V_{p} \sin 2 \pi f t\right)$, the rate of change at any time in the cycle is given by $\frac{d V_{0}}{d t}=2 \pi f V_{p} \cos 2 \pi f t$. The slope of this signal will have its maximum amplitude when the cosine term becomes $\pm 1$.

$$
f_{\max }=\frac{S R}{2 \pi V_{p}}
$$

$$
\begin{aligned}
& f_{\max }= \text { maximum frequency without } \\
& \text { slew rate distortion }[\mathrm{Hz}] \\
& S R= \text { output voltage rate of change } \\
& {[\mathrm{V} / \mathrm{s}] } \\
& V_{p}= \text { peak output voltage }[\mathrm{V}]
\end{aligned}
$$

## $f_{H f}$ SMALL SIGNAL BANDWIDTH

Since the gain bandwidth product remains constant, the application of feedback reduces the gain and increases the bandwidth by the same factor of $\left(1+\beta A_{0}\right)$.

$$
f_{H f}=\left(1+\beta A_{0}\right) f_{H}
$$

where $\beta=\frac{R_{1}}{R_{1}+R_{f}}$
(Add $R_{g}$ to $R_{1}$ if present.)
$f_{H f}=$ small signal bandwidth, or high frequency corner with feedback [Hz]

$$
\beta=b \text { box gain }
$$

$A_{0}=$ open loop gain in the passband
$f_{H}=f_{U} / A_{0}$ high freq. corner [Hz]

## $V_{i N}$ NORMALIZED INPUT VOLTAGE

The input voltage is normalized to unity to use in plotting normalized emitter/drain currents versus normalized input voltages in a differential amplifier.
$V_{i N}=\frac{V_{i}}{V_{P}} \sqrt{\frac{I_{D S S}}{I_{Q}}}$
$V_{i}=$ differential input voltage [V]
$V_{P}=$ peak output voltage [V]
$I_{D S S}=$ drain to source scale current or saturation current? [A]
$I_{Q}=$ bias current [A]

## EMITTER COUPLED LOGIC

With $v_{R}$ taken to be a reference, the circuit below functions as an inverter with input $v_{I}$ and output $v_{01}$. The advantage to this circuit is that the collector currents (one or the other) become saturated without much charge on the base, permitting fast operation
$i_{C 1}=I_{S} e^{\left(v_{I}-v_{E}\right) / V_{T}}$
$i_{C 2}=I_{S} e^{\left(v_{R}-v_{E}\right) / V_{T}}$ where $V_{T}=\frac{k T}{q}$
$I_{S}=$ scale current [A]
$I_{Q}=$ bias current $[\mathrm{A}]$


JFETs

$Y_{f s}$ FORWARD TRANSCONDUCTANCE

$$
Y_{f s} \equiv \frac{\partial I_{D}}{\partial V_{G S}} \quad Y_{f s Q}=\left|\frac{2 I_{D S S}}{V_{P}}\right|\left[1-\frac{V_{G S}}{V_{p}}\right]
$$

$Y_{f s}=$ slope of the FET transfer characteristic [A]
$I_{D S S}=$ scale current or saturation current? [A]
$V_{G S}=$ gate-to-source voltage [V]
$V_{P}=$ pinchoff voltage, the highest voltage at which there is still no drain current [V]

## $I_{D}$ DRAIN CURRENT <br> $I_{D}=I_{D S S}\left[1-\frac{V_{G S}}{V_{p}}\right]^{2}$

$I_{D S S}=$ scale current or saturation current? $[\mathrm{A}]$
$V_{G S}=$ gate-to-source voltage [V]
$V_{P}=$ pinchoff voltage, the highest voltage at which there is still no drain current [V]

## $g_{f \max }$ JFET MAXIMUM FORWARD TRANSCONDUCTANCE

$I_{D S S}=$ drain to source scale current or saturation current? [A]
$g_{f \max }=\frac{\sqrt{I_{D S S} I_{Q} / 2}}{\left|V_{P}\right|}$
$I_{Q}=$ differential amplifier supply current [A]
$V_{P}=$ pinchoff voltage [V]
$Y_{o s}$ JFET OUTPUT ADMITTANCE

$$
Y_{o s} \equiv \frac{\partial I_{D}}{\partial V_{D S}} \quad Y_{o s}=\frac{1}{r_{o}}=\frac{I_{D}}{V_{A}}
$$

$Y_{o s}=$ slope of the output characteristic $\left[\Omega^{-1}\right]$
$I_{D}=$ drain current [A]
$V_{D S}=$ drain to source voltage [V]
$V_{A}=$ Early voltage, typically 100 V [V]


DIFFERENTIAL INPUT GAIN (with FETs)

$$
A_{d}=\frac{2 g_{f \max }}{g_{2}+g_{4}}=\frac{\frac{2 \sqrt{I_{D S S} I_{Q} / 2}}{\left|V_{P}\right|}}{4 V_{T}} \div\left(\frac{I_{Q} / 2}{V_{A J F E T}}+0\right)
$$

Harmonic
distortion is caused by the non-linearity of the transfer characterictic.

$V_{\text {out }}$ has the form
$V_{\text {out }}=A_{0}+A_{1} \sin \left(\omega t+\phi_{1}\right)+A_{2} \sin \left(2 \omega t+\phi_{2}\right)+\cdots$
The percent second harmonic distortion would be

$$
D_{2 H}=\frac{A_{2}}{A_{1}} \times 100
$$

For the JFET, since the drain current is proportional to the output voltage, substitute

$$
V_{G S}=V_{G S Q}+V_{\text {in }} \sin \omega t \text { into } I_{D}=I_{D S S}\left[1-\frac{V_{G S}}{V_{p}}\right]^{2}
$$

We are only interested in the terms formed by the values within the parenthesis. Use the trig identity

$$
\sin ^{2} \omega t=\frac{1}{2}-\frac{1}{2} \cos 2 \omega
$$

(It doesn't matter, for distortion purposes, whether it's sine or cosine.)

## MOSFETs

## MOSFET INPUT STAGE CHARACTERISTICS

- very low input current (a few pA)
- very high $Z_{\text {in }}$ (up to $10^{15} \Omega$ )
- reasonable $f_{u}$ and slew rate
- high offset voltages (tens of mV )
- high noise
- vulnerable to static electricity

$i_{D}$ MOSFET DRAIN CURRENT
NMOS
triode region

$$
i_{D n}=\frac{1}{2} k_{n}^{\prime}\left(\frac{W}{L}\right)_{n}\left[2\left(v_{G S}-V_{t n}\right)-v_{D S}\right] V_{D S}
$$

NMOS
saturation
$i_{D n}=\frac{1}{2} k_{n}^{\prime}\left(\frac{W}{L}\right)_{n}\left(v_{G S}-V_{t n}\right)^{2}$
PMOS
triode region

$$
i_{D p}=\frac{1}{2} k_{p}^{\prime}\left(\frac{W}{L}\right)_{p}\left[2\left(v_{S G}-\left|V_{t p}\right|\right)-v_{S D}\right] V_{S D}
$$

PMOS
saturation

$$
i_{D p}=\frac{1}{2} k_{p}^{\prime}\left(\frac{W}{L}\right)_{p}\left(v_{S G}-\left|V_{t p}\right|\right)^{2}
$$

$k^{\prime}=$ process transconductance parameter $\left[\mathrm{mA} / \mathrm{V}^{2}\right]$
$W / L=$ channel width to length ratio
$v_{G S}=$ gate-to-source voltage [V]
$V_{t}, V_{t p}=$ threshhold voltage [V]
$v_{D S}=$ drain-to-source voltage [ V ]
$v_{S D}=$ source-to-drain voltage [V]
Point of Confusion: An NMOS transistor leaves the saturation region and enters the triode region when the gate voltage has reached a level such that further increases do not produce additional drain current. This is essentially the opposite of the definition for saturation current in a BJT. For the NMOS transistor we have:

$$
\begin{array}{cc}
\text { triode region } & \text { saturation region } \\
v_{G S}>v_{D S}+V_{t} & v_{G S} \leq v_{D S}+V_{t} \\
\hline
\end{array}
$$

## $g_{f \max }$ MOSFET MAXIMUM FORWARD TRANSCONDUCTANCE

$$
g_{f \max }=\sqrt{\frac{K I_{Q}}{2}}
$$

$K=$ process transconductance parameter $\left[\mathrm{mA} / \mathrm{V}^{2}\right.$ ]
$I_{Q}=$ differential amplifier supply
current [A]
$A_{1}$ MOSFET INPUT STAGE GAIN $A_{1}=\frac{2 g_{f \max }}{g_{\text {total }}}=\frac{2 g_{f \text { max }}}{g_{2}+g_{5}}=2 \sqrt{\frac{K I_{Q}}{2}} \div\left(\frac{I_{Q}}{2 V_{A \text { moset }}}+\frac{I_{B 5}}{V_{T}}\right)$

## DEPLETION-TYPE MOSFET

An n-channel (NMOS) depletion-type MOSFET already has a channel formed with no gate voltage applied. To turn off the transistor, a negative gate voltage is applied. So the threshold voltage is negative. The enhancement-type NMOS transistor requires a positive gate voltage for the device to conduct.
NMOS depletiontype:


NMOS enhancementtype:

## CMOS



## MOSFET SATURATION

This is much different from BJT saturation as it occurs when the gate voltage is low. The MOSFET is in saturation when the gate voltage is less than the sum of the drain-to-source voltage and the threshold voltage.

\[

\]

## NOISE

## NOISE - Any unwanted signal

Electronic noise is the type of noise we will study. It can be minimized but not eliminated.

Noise referral is the practice of quantifying by determining the level of noise at the input that would produce the amount of noise in question. (refer to document NoiseReferral.pdf)

Shot noise (Schottky noise) is due to a random number of carriers participating in current flow across a p-n junction. This is a white noise, meaning it has a uniform power spectral density (noise power per hertz). Base current has a shot noise component.
Thermal noise or Johnson noise occurs in resistors and is due to the thermal aggitation of free electrons in the resistor material. Net movement is zero but at times electrons are moving one way or the other. Thermal noise is also white noise.

Excess noise or flicker noise occurs in resistive material when a DC current flows through it. It is in excess of thermal noise.

## $i_{n}$ NOISE COMPONENT and BANDWIDTH

The noise component of an average current $I_{Q}$ is given by
$i_{n}=\sqrt{2 q I_{Q} B}$
$i_{n}=$ noise $[\mathbf{A} / \sqrt{\mathbf{H z}}]$
$q=$ electron charge $1.602 \times 10^{-19} \mathrm{C}$
$I_{Q}=$ differential amplifier supply current [A]
$B=\frac{\pi}{2} f_{H}$
$B=$ the subsequent noise bandwidth which follows in the amplifier [Hz]

## $V_{n}$ THERMAL NOISE

Occurs in resistors due to the thermal agitation of free electrons
$V_{n}=\sqrt{4 k T R B}$
$V_{n}=$ thermal noise [ $\mathrm{V}_{\text {rms }}$ ]
$k=$ Boltzman's constant $1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K}$
$T$ = temperature [K]
$R=$ resistance $[\Omega]$
$B=$ the subsequent noise bandwidth which follows in the amplifier [Hz]

## $V_{n i}$ TOTAL BJT NOISE IN A DIFFERENTIAL PAIR

Occurs in BJT transistors due to thermal noise in the base spreading resistance and shot noise in the collector current (referred to the input).

$$
V_{n i}=\sqrt{\underbrace{8 k T r_{b b^{\prime}}}_{\text {base spreading }}+\underbrace{16 V_{T}^{2}\left(q / \alpha I_{Q}\right)}_{\text {shot noise referred to input }}}
$$

$V_{n i}=(\mathrm{BJT})$ differential input noise $[\mathbf{v} / \sqrt{\mathbf{H z}}]$
$r_{b b^{\prime}}=r_{x}$ base spreading resistance $[\Omega]$

## $V_{n i}$ FET INPUT NOISE

Occurs in FET transistors due thermal noise in the channel

$V_{n i}=(\mathrm{FET})$ input noise $[\mathbf{V} / \sqrt{\mathbf{H z}}]$
$Y_{f s}=$ maximum forward transconductance [A/V]

## $i_{e x}$ EXCESS NOISE

Excess noise or flicker noise occurs in resistive material when a DC current flows through it. It is in excess of thermal noise. It is dependent on the type of resistive material and varies with frequency. It is normally measured (not calculated).
$i_{e x}=\sqrt{\frac{K I_{D C} B}{f}} \quad \begin{aligned} & K=\text { a constant, dependent on material } \\ & I_{D C}=\text { current in the resistor [A] } \\ & B=\text { the subsequent noise bandwidth } \\ & \text { (which follows in the amplifier [Hz }\end{aligned}$

## NF NOISE FIGURE

The ratio of total noise (referred to the input) to the noise due to the impedance of the input device $\left(\mathrm{R}_{\mathrm{g}}\right)$.

$$
\begin{aligned}
& N F=\text { noise figure }[\mathrm{dB}] \\
& V_{a}^{2}=\text { equivalent input noise } \\
& \text { squared }\left[\mathrm{V}^{2} / \mathrm{Hz}\right] \\
& k=\text { Boltzman's constant } \\
& 1.38 \times 10^{-23} \mathrm{~J} / \mathrm{K} \\
& T=\text { temperature }[\mathrm{K}] \\
& R_{g}=\text { source impedance }[\Omega]
\end{aligned}
$$

## $V_{n o}$ NOISE VOLTAGE AT THE OUTPUT

The noise level at the output, assuming uniform spectral noise density and given the noise bandwidth.

$$
V_{n o}=V_{a} \sqrt{B}
$$

$V_{n o}=$ noise voltage at the output [V]
$V_{a}=$ equivalent input noise squared [ $\mathrm{V} / \sqrt{\mathrm{Hz}}$ ]
$B=$ noise bandwidth [ Hz ]

## SNR SIGNAL TO NOISE RATIO

$S N R=20 \log _{10} \frac{V_{S}}{V_{N}}$
$N F=S N R_{i}-S N R_{o}$

## NOISE REFERRAL

All noise in the amplifier system is converted to an equivalent input noise, i.e. the level of noise at the input which would result in the level of noise under study. The example below shows how to refer the noise in resistor $R_{1}$ to an equivalent input noise.
$V_{\text {neqR1 }}$ is multiplied by the gain of the amp to give the noise at the output. Likewise, $V_{n R 1}$ (the noise in $R_{1}$ ) sets up a current through $R_{1}$ and goes to ground at the op amp input (asymetric conditions).
Since this
 current cannot flow into the op amp, it flows through $R_{\mathrm{f}}$, developing a voltage at the output. Although this voltage will be negative, it may be considered positive since it will be squared later. Refer to Thermal Noise to calculate $V_{n R 1}$.

## VOLTAGE OFFSET

## FINDING THE VOLTAGE OFFSET

It is desired that the output voltage be zero in the absence of an input signal. Factors conspiring against this are rated in terms of

$$
\begin{aligned}
& \boldsymbol{V}_{o s} \text { - input offset voltage, } \\
& \boldsymbol{I}_{\boldsymbol{B}} \text { - input bias current }\left(I_{B-}+I_{B+}\right) / 2 \\
& \boldsymbol{I}_{\boldsymbol{O}} \text { - input offset current }\left|I_{B-}-I_{B+}\right|
\end{aligned}
$$

To find the resulting output voltage offset, a model is created and the effect of the three inputs, $V_{o s+}, I_{B+}$, \& $I_{B}$, on the output is calculated. To find the worst case, the offset voltage source and the larger of the two current sources are applied to the input where they will do the most harm. For both the inverting and non-inverting amplifiers, this seems to be the positive input, provided that it is not tied to ground.

## SINGLE-SUPPLY AMPLIFIERS

## INVERTING

The inverting amplifier has less noise than the non-inverting amplifier due to the lower value (1k) of resistors that bring the DC input level to mid-supply, and the fact that the capacitor connected to their midpoint shunts noise to ground. The main consideration in sizing these would be current drain.


## NON-INVERTING

This amplifier is very noisy due to the two $1 \mathrm{M} \Omega$ resistors that are used to bring the DC input level to mid-supply. Reducing the value of these resistors would lower the input impedance.


## NON-INVERTING OFFSET MODEL



The offset voltage is equal to the voltage at the positive input multiplied by the gain, added to the voltage at the output resulting from current $I_{B}$. For the worst case offset for this amplifier, let $I_{B+}=I_{B-}+I_{O}$.

$$
V_{\text {out offset }}=\frac{R_{f}+R_{1}}{R_{1}}\left(V_{o x}+I_{B+} R_{g}\right)-I_{B-} R_{f}
$$

## DIFFERENTIAL OP-AMP AMPLIFIERS

## SINGLE OP-AMP DIFFERENTIAL AMPLIFIER

This is not a true differential amplifier since the output is a weighted difference, unless we set $R_{f} / R_{1}=R_{g} / R_{2}$. When calculating differential gain, $V_{1}=V_{i n} / 2$ and $V_{2}=-V_{i n} / 2$.


By superposition, the output is $V_{\text {out V1 }}+V_{\text {out V2 }}$.

Differential gain: $A_{\text {diff }}=\frac{V_{\text {out } V 2}-V_{\text {out } V 1}}{V_{2}-V_{1}}=\frac{A_{2}-A_{1}}{2}$
For common mode gain, $V_{1}=V_{2}=V_{\mathrm{cm}}$ and

$$
A_{c m}=\frac{V_{1 \text { out } c m}+V_{2 \text { out } c m}}{V_{c m}}=A_{1}+A_{2}
$$

INSTRUMENTATION AMPLIFIER
This type of amplifier has high gain ( $10^{5}$ or so) and high common mode rejection ratio (>100 dB) for the amplification of very weak signals. Gain can be controlled by the value of $R_{1}$.


$$
A_{\text {diff }}=\frac{R_{4}}{R_{3}}\left(1+2 \frac{R_{2}}{R_{1}}\right)
$$

## FILTERS



## LEAKY INTEGRATOR

The leaky integrator is a $\mathbf{1}^{\text {st }}$ order low-pass filter.
The order of a filter refers to the order of the polynomial in the denominator of the transfer function and usually corresponds to the number of frequency elements, in this case, one capacitor.

At low frequencies, feedback passes (leaks) through $R_{f}$ and at high frequencies feedback passes through $C_{f}$.


$$
\begin{aligned}
\frac{V_{\text {out }}}{V_{\text {in }}}(j \omega)=-\frac{Z_{f}}{Z_{1}} & =-\frac{\frac{1}{j \omega C_{f}} \| R_{f}}{R_{1}}=\frac{-\frac{R_{f}}{R_{1}}}{1+j \omega C_{f} R_{f}} \\
\frac{V_{\text {out }}}{V_{\text {in }}}(j \omega) & =\frac{-\frac{R_{f}}{R_{1}}}{1+j \frac{f}{f_{H}}} \quad f_{H}=\frac{1}{2 \pi C_{f} R_{f}}
\end{aligned}
$$

## DIFFERENTIATOR

There is lagging phase shift in addition to the op amp's phase shift, so phase reversal can occur when loop gain is less than 1.

$V_{\text {out }}=-i_{f} R_{f}=-R_{f} C \frac{d v_{\text {in }}}{d t} \quad f_{C}=\sqrt{f_{B} f_{u}}$


Gain: $\frac{V_{\text {out }}}{V_{\text {in }}}(j \omega)=-\frac{Z_{f}}{Z_{1}}=-\frac{R_{f}}{\frac{1}{j \omega C}}=-j 2 \pi f C R_{f}$
Let $f_{B}=\frac{1}{2 \pi C R_{f}} \quad \frac{V_{\text {out }}}{V_{\text {in }}}(j \omega)=-j \frac{f}{f_{B}}$
gain $(d B)=20 \log _{10} f-20 \log _{10} f_{B}$

FIRST ORDER HIGH PASS FILTER
There is lagging phase shift in addition to the op amp's phase shift, so phase reversal can occur when loop gain is less than 1.


## FIRST ORDER BAND PASS FILTER


$f_{L}=\frac{1}{2 \pi C_{1} R_{1}}$
$f_{H}=\frac{1}{2 \pi C_{f} R_{f}}$

$$
\frac{V_{\text {out }}}{V_{\text {in }}}(j \omega)=-\frac{Z_{f}}{Z_{1}}=-\frac{\left.\frac{1}{j \omega C_{f}} \right\rvert\, R_{f}}{R_{1}+\frac{1}{j \omega C_{1}}}=\frac{-\frac{R_{f}}{R_{1}}}{\left(1-j \frac{1}{\omega C_{1} R_{1}}\right)\left(1+j \omega C_{f} R_{f}\right)}
$$

$$
=\frac{-\frac{R_{f}}{R_{1}}}{\left(1-j \frac{f_{L}}{f}\right)\left(1+j \frac{f}{f_{H}}\right)}
$$



## OSCILLATORS



Find $V_{1}$ and $V_{2}: V_{1}=V_{\text {in }} \frac{R_{5}}{R_{3}+R_{4}+R_{5}}$

$$
V_{2}=V_{\text {in }} \frac{\frac{R \times \frac{1}{j \omega C}}{R+\frac{1}{j \omega C}}}{R+\frac{1}{j \omega C}+\frac{R \times \frac{1}{j \omega C}}{R+\frac{1}{j \omega C}}}=V_{\text {in }} \frac{R}{3 R+j\left(\omega C R^{2}-\frac{1}{\omega C}\right)}
$$

$V_{2}$ (and $V_{\text {out }}$ ) will be in phase with $V_{\text {in }}$ when
$\omega C R^{2}=1 /(\omega C)$ so
$\omega_{0}{ }^{2}=\frac{1}{R^{2} C^{2}}, \omega_{0}=\frac{1}{R C}$ and $f_{0}=\frac{1}{2 \pi R C}$
At $\omega_{0}, V_{1}=V_{\text {in }} \frac{R_{5}}{R_{3}+R_{4}+R_{5}}$ and $V_{2}=\frac{1}{3} V_{\text {in }}$
The loop gain is the $1^{\text {st }}$ stage gain $\times 2^{\text {nd }}$ stage gain:
$\frac{V_{2}-V_{1}}{V_{\text {in }}} \times \frac{V_{\text {out }}}{V_{2}-V_{1}}=\left(\frac{1}{3}-\frac{R_{5}}{R_{3}+R_{4}+R_{5}}\right) \frac{R_{f}}{R_{1}}$
To just sustain oscillations, the loop gain must be 1:
$\frac{R_{1}}{R_{f}}=\frac{1}{3}-\frac{R_{5}}{R_{3}+R_{4}+R_{5}}$
$R_{3}$ is a varistor which serves to limit the gain.

## FUNCTION GENERATOR

This function generator produces a square wave at $V_{A}$, and a triangle wave at $V_{B}$. Due to the positive feedback in the $1^{\text {st }} \mathrm{IC}, V_{A}$ will go to one of the rails. If $V_{A}$ is positive, then to get it to go negative, $V_{i n+}$ needs to be slightly negative.


$$
V_{B}=-\frac{k V_{\text {RALL }}}{R_{3} C} t
$$

Frequency of oscillation:

$$
f_{0}=\frac{k R_{2}}{4 R_{1} R_{3} C}
$$

Square wave time to rise:


$$
t_{R}=0.8 \frac{2 V_{R A L L}}{S R}
$$

## ASTABLE MULTIVIBRATOR

$$
\begin{gathered}
k=\frac{R_{1}}{R_{1}+R_{2}} \\
\tau=R_{3} C
\end{gathered}
$$



Frequency of oscillation:
$f_{0}=\frac{1}{2 R_{3} C \ln \left(\frac{1+k}{1-k}\right)}$
Rise time is the time it takes to get from $10 \%$ to $90 \%$ of asymptote and is dependent on the slew rate $S R$.
rise time $=\frac{2 V_{R A L}}{S R} \times 0.8$


## GENERAL

## CHARACTERISTICS OF THE IDEAL OP AMP

- The difference between the voltages at the inputs $\left(v_{2}-v_{1}\right)$ multiplied by the open-loop gain $A$ yields the op amp output $A\left(v_{2}-v_{1}\right)$.
- The input impedance is infinite.
- The input current is zero.
- The output impedance is zero.
- The output current is whatever is required to maintain the output voltage.
- The output is in phase with the signal at the positive input.
- Infinite common-mode rejection, the rejection of identical signals at the + and - inputs.
- The open-loop gain $A$ is equal for all frequencies.
- The open-loop gain $A$ is infinite.


## GRAPHING TERMINOLOGY

With $x$ being the horizontal axis and $y$ the vertical, we have a graph of $\boldsymbol{y}$ versus $\boldsymbol{x}$ or $\boldsymbol{y}$ as a function of $\boldsymbol{x}$. The $x$-axis represents the independent variable and the $y$-axis represents the dependent variable, so that when a graph is used to illustrate data, the data of regular interval (often this is time) is plotted on the $x$-axis and the corresponding data is dependent on those values and is plotted on the $y$ axis.

## MILLER THEOREM

The circuit at left may be replaced by the circuit at right. (One of the resistances in the circuit at right will actually be negative).


For an admittance $Y$ we have:


## DECIBEL CONVERSION

$$
\begin{array}{ll}
G_{d B}=20 \log _{10} G_{V / V} & G_{d B}=\text { Gain in decibels } \\
G & G_{V / V}=\text { Gain in volts per volt }
\end{array}
$$

