TWO-DIMENSIONAL SYSTEMS

Method for solving a two-dimensional first order system of differential equations

from section 4.1 p228.

13. p228 x' = -2y x(0) = 1 Given a system of differential equations and initial conditions, y' = 2x y(0) = 0 Find the general solutions and the particular solutions.

Take the derivative of the first expressionx'' = -2y'Substitute 2x for y'x'' = -2(2x)Now solve the resulting second order equationx'' + 4x = 0

The characteristic equation is $r^2 + 4 = 0$ which has complex roots $r = 0 \pm 2i$ This gives us the general solution $x(t) = e^{0t} (A \cos 2t + B \sin 2t)$

Taking the derivative gives $x'(t) = -2A \sin 2t + 2B \cos 2t$

The first problem expression can be rewritten $y = \frac{x'}{-2}$ so that $y(t) = -\frac{1}{2}x'(t)$ by substituting for x'(t) we have $y(t) = -\frac{1}{2}(-2A\sin 2t + 2B\cos 2t)$ which reduces to $y(t) = A\sin 2t + B\cos 2t$

we now have this general solution to the system of equations to which we can apply the initial conditions $x(t) = (A\cos 2t + B\sin 2t)$ $y(t) = A\sin 2t + B\cos 2t$

General solutions: $x(t) = A \cos 2t + B \sin 2t$ $y(t) = A \sin 2t + B \cos 2t$ Initial conditions:x(0) = 1y(0) = 0substituting: $1 = A \cos 0 + B \sin 0$ $0 = A \sin 0 + B \cos 0$ determines the constants:A = 1B = 0

This yields the particular solutions: $x(t) = \cos 2t$ $y(t) = \sin 2t$

Tom Penick tomzap@eden.com October 24, 1997