## TWO-DIMENSIONAL SYSTEMS

Method for solving a two-dimensional first order system of differential equations from section 4.1 p228.
13. p228

| $x^{\prime}=-2 y$ | $x(0)=1$ |
| :---: | :---: |
| $y^{\prime}=2 x$ | $y(0)=0$ |

Given a system of differential equations and initial conditions, find the general solutions and the particular solutions.

Take the derivative of the first expression

$$
x^{\prime \prime}=-2 y^{\prime}
$$

Substitute $2 x$ for $y^{\prime}$

$$
x^{\prime \prime}=-2(2 x)
$$

Now solve the resulting second order equation

$$
x^{\prime \prime}+4 x=0
$$

The characteristic equation is $r^{2}+4=0 \quad$ which has complex roots $\quad r=0 \pm 2 i$
This gives us the general solution $x(t)=e^{0 t}(A \cos 2 t+B \sin 2 t)$

Taking the derivative gives $\quad x^{\prime}(t)=-2 A \sin 2 t+2 B \cos 2 t$

The first problem expression can be rewritten $\quad y=\frac{x^{\prime}}{-2} \quad$ so that $\quad y(t)=-\frac{1}{2} x^{\prime}(t)$
by substituting for $x^{\prime}(t)$ we have $y(t)=-\frac{1}{2}(-2 A \sin 2 t+2 B \cos 2 t)$
which reduces to $\quad y(t)=A \sin 2 t+B \cos 2 t$
we now have this general solution to the system of equations to which we can apply the initial conditions

$$
\begin{aligned}
& x(t)=(A \cos 2 t+B \sin 2 t) \\
& y(t)=A \sin 2 t+B \cos 2 t
\end{aligned}
$$

| General solutions: | $x(t)=A \cos 2 t+B \sin 2 t$ | $y(t)=A \sin 2 t+B \cos 2 t$ |
| :--- | :--- | :--- |
| Initial conditions: | $x(0)=1$ | $y(0)=0$ |
| substituting: | $1=A \cos 0+B \sin 0$ | $0=A \sin 0+B \cos 0$ |
| determines the constants: | $A=1$ | $B=0$ |

This yields the particular solutions:

$$
\begin{aligned}
& x(t)=\cos 2 t \\
& y(t)=\sin 2 t
\end{aligned}
$$

