DC CIRCUITS

Ohm's Law: *volts = amps × ohms* $E = IR$

Power: *watts = volts × amps* $P = EI$

watts = volts² / ohms $P = \frac{E^2}{R}$

watts = amps² × ohms $P = I^2 R$

Coulomb: *coulombs = amps × seconds* $Q = It$

Kirchhoff's Voltage Law: In any closed electric circuit, the algebraic sum of the voltage drops must equal the algebraic sum of the applied emfs.

Voltage Divider Theorem: In a series circuit, the portion of applied emf developed across each resistor is in the ratio of that resistor's value to the total series resistance.

The applied emf is divided up between the series resistors. The voltage across each of two resistors can be calculated by:

$$V_1 = \frac{ER_1}{R_1 + R_2} \qquad V_2 = \frac{ER_2}{R_1 + R_2}$$

where E is the supply voltage.

Kirchhoff's Current Law: The algebraic sum of the currents entering a point in an electric circuit must equal the algebraic sum of the currents leaving that point.

When two resistors are in parallel: (This may be referred to as a current divider.)

$$I_1 = \frac{E}{R_1} \qquad I_2 = \frac{E}{R_2} \qquad I = I_1 + I_2$$

For multiple resistors in parallel, the current is:

$$I = E\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}\right)$$

For multiple resistors in parallel, the equivalent resistance is:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

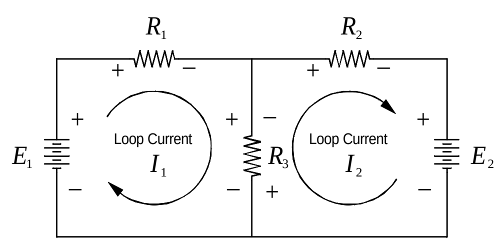
Network analysis using Kirchoff's Laws: The procedure is as follows:

1. Letter all junctions on the network *A, B, C,* etc.
2. Identify current directions and voltage polarities, and number them according to the resistor involved.
3. Identify each current path according to the lettered junctions and, applying Kirchhoff's voltage law, write the voltage equations for the paths. i.e. for the path *ABCDE*, $E_1 = I_1 R_1 + I_2 R_2$
4. Applying Kirchhoff's current law, write the equations for the currents entering and leaving all junctions where more than one current is involved. (as shown above)
5. Solve the equations by substitution to find the unknown currents. (I_1, I_2, etc.)

Note that in some circumstances currents and voltage polarities will turn out to be negative when the circuit is analyzed. This simply means that the assumed current directions and voltage polarities were incorrect.

Network Analysis using Loop Equations: The procedure is as follows:

1. Draw all loop currents in a clockwise direction and identify them by number. i.e. I_1, I_2, etc.
2. Identify all resistor voltage drops as + to - in the direction of the loop current. (Sometimes there may be voltage components in both directions.)
3. Identify all voltage sources according to their correct polarity. (The voltage loop may not be in the same direction as current flow.)
4. Write the equations for the voltage drops around each loop in turn, by equating the sum of the voltage drops to zero.
5. Solve the equations to find the unknown currents.



Nodal Analysis: A *voltage node* is a junction in an electrical circuit at which a voltage can be measured with respect to another (reference) node. The procedure for nodal analysis is as follows:

1. Convert all voltage sources to current sources, and redraw the circuit.
2. Identify all nodes and choose a reference node.
3. Write the equations for the currents flowing into and out of each node, with the exception of the reference node.
4. Solve the equations to determine the node voltages and the required branch currents.

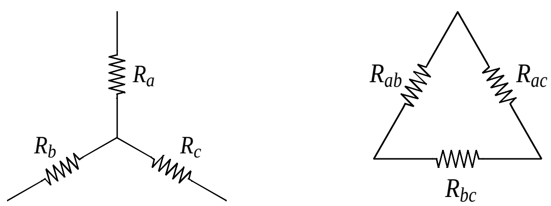
Internal resistance is the characteristic of all voltage sources that tends to reduce the voltage and current they can deliver under load.

$$R_i = \frac{V_{NL} - V_{FL}}{I_L}$$

where: V_{NL} is the noload voltage
V_{FL} is the full load voltage
I_L is the amperage under load

With R_i known, the voltage source circuit can be represented by an ideal voltage source in series with a resistor R_i.

Delta-Wye Transformation:



$$R_a = \frac{R_{ab}R_{ac}}{R_{ab} + R_{ac} + R_{bc}} \qquad R_{ab} = \frac{R_aR_b + R_aR_c + R_bR_c}{R_c}$$

$$R_b = \frac{R_{ab}R_{bc}}{R_{ab} + R_{ac} + R_{bc}} \qquad R_{ac} = \frac{R_aR_b + R_aR_c + R_bR_c}{R_b}$$

$$R_c = \frac{R_{ac}R_{bc}}{R_{ab} + R_{ac} + R_{bc}} \qquad R_{bc} = \frac{R_aR_b + R_aR_c + R_bR_c}{R_a}$$

Superposition Theorem: In a network containing more than one source of voltage or current, the current through any branch is the algebraic sum of the currents produced by each source acting independently.

Procedure: 1) Select one source, and replace all other sources with their internal impedances; 2) Determine the level and direction of the current that flows through the desired branch as a result of the single source acting alone; 3) Repeat steps 1 and 2 using each source in turn until the branch current components have been calculated for all sources; 4) Sum the component currents to obtain the actual branch current.

Thévenin's Theorem: Any two-terminal network containing resistances and voltage sources and/or current sources may be replaced by a single voltage source in series with a single resistance. The emf of the voltage source is the open-circuit emg at the network terminals, and the series resistance is the resistance between the network terminals when all sources are replaced by their internal impedances.

Procedure: 1) Calculate the open-circuit terminal voltage of the network; 2) Redraw the network with each voltage source replaced by a short circuit in series with its internal resistance, and each current source replaced by abn open circuit in parallel with its internal resistance; 3) Calculate the resistance of the redrawn network as *seen* from output terminals.

Norton's Theorem: Any two-terminal network containing resistances and voltage source and/or current sources may be replaced by a single current source in parallel with a single resistance. The output from the current source is the short-circuit current at the network terminals, and the parallel resistance is the resistance between the network terminals when all sources are replaced by their internal impedances.

Procedure: 1) Calculate the short-circuit current at the network terminals; 2) Redraw the network with each voltage source replaced by a short-circuit in series with its internal resistance, and each current source replaced by an open circuit in parallel with its internal resistance; 3) Calculate the resistance of the redrawn network as *seen* from the output terminals.

Millman's Theorem: Multiple current sources in parallel can be represented by a single current generator having the sum of the individual source currents and the resistance of the parallel combination of the individual source resistances.

Maximum Power Transfer Theorem: Maximum output power is obtained from a source when the load resistance is equal to the output resistance of the network or source as seen from the terminals of the load.

A voltage source having a voltage E and a source resistance R_S can be replaced by a *current source* with a current E/R_S and a source resistance R_S.

A current source having a current I and a source resistance R_S can be replaced by a *voltage source* with a voltage IR_S and a source resistance R_S.

Conductance: The reciprocal of resistance in units of *siemens* (S). For multiple resistors in parallel, the conductances are:

$$G = G_1 + G_2 + G_3 + \cdots + G_n$$

CAPACITANCE

The **farad** is the SI unit of capacitance, equal to the capacitance of a capacitor that contains a charge of 1 coulomb when the potential difference between its terminals is 1 volt.

$Q = CE$ where Q = the charge in coulombs
C = capacitance in farads
E = voltage across the capacitor

The dielectric is the insulating material between the conducting plates. After a capacitor is discharged, a small charge may remain due to polarized atoms in the dielectric; this phenomenon is known as dielectric absorption.

Permittivity is the ease with which electric flux is permitted to pass through a given dielectric material.

Dielectric constant is the term for relative permittivity $\in_0$.

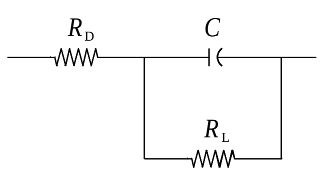
$$C = \frac{\in_r \in_o A}{d}$$

where $\in_r$ = relative permittivity
$\in_o$ = permittivity of free space
A = plate area in m^2
d = dielectric thickness in m

Some Dielectric Constants $\in_r$

Vacuum	1	Glass	5-100	Paper	4-6
Air	1.006	Mica	3-7	Polystyrene	2.5
Ceramic, low loss	6-20	Mylar	3	Teflon	2
Ceramic, hi $\in_r$	>1000	Oxide Film	5-25		

Capacitor equivalent circuit



R_D = resistance of the plates
R_L = leakage resistance of the dielectric

Air Capacitors are usually variable type.

Paper Capacitors consist of layers of paper and metal foil or just metalized paper that is rolled up, dipped, and terminated. A band on one end may indicate the *outside* foil so that it may be grounded for shielding, not necessarily polarity. Values range from 500 pF to 50 µF to 600 V. Lowest cost but physically larger.

Plastic Film Capacitors are similar to paper capacitors but use polystyrene or Mylar instead of paper. Insulation resistance is greater than 100 000 MΩ. Values typically range from 5 pF to 0.47 µF to 600 V.

Mica Capacitors consist of layers of mica and metal foil or layers of silvered mica. Precise values and wide temperature ranges are possible. Values typically range from 1 pF to 0.1 µF to 35 000 V.

Ceramic Capacitors consist of a ceramic disc with films of metal on both sides. There are two types of ceramic material; one with extremely high permittivity but low leakage resistance which allows smaller physical size, and the other has lower permittivity and leakage resistances of about 7500 MΩ but with large physical size.

Electrolytic Capacitors are constructed with two sheets of aluminum foil separated by a fine gauze soaked in electrolyte and rolled up and encased in an aluminum cylinder. When assumbled, a direct voltage is applied to the capacitor terminals causing a thin layer of aluminum oxide to form on the surface of the positive plate next to the electrolyte. The aluminum oxide is the dielectric and the electrolyte and positive sheet of foil are the capacitor plates. High capacitances in a relatively small size are obtained. Working voltages are low and leakage current is high. The capacitors are polarized, and if connected incorrectly, gas forms within the electrolyte and the capacitor may explode. Nonpolorized electrolytic capacitors are available which consist of two capacitors in one package connected back to back.

Tantalum Capacitors are essentially another type of electrolytic capacitor. Tantalum is *sintered* (baked) into a porous solid. This is immersed into a container of electrolyte which is then absorbed into it.

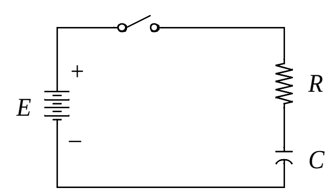
Capacitors in Series:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}$$

Capacitors in Parallel: The total capacitance is the sum of the individual capacitances:

$$C = C_1 + C_2 + C_3 + \ldots + C_n$$

Voltage on a Capacitor:



$$e_C = E - (E - E_0)2.718^{-t(CR)} \quad \text{or}$$

$$\frac{t}{CR} = \ln\left(\frac{E}{E - e_C}\right)$$

where e_C = capacitor voltage at time t
E = supply voltage
E_0 = initial level of capacitor voltage
t = time in seconds
C = capacitance in farads
R = resistance (series) in ohms

MAGNETISM

Flux: magnetic lines of force which form closed loops.

Right-hand rule: When the thumb points in the direction of current flow, the fingers show the direction of the magnetic lines of force around a conductor. When a *solenoid* is gripped with the right hand such that the fingers are pointing in the direction of current flow in the coils, the thumb points in the direction of the flux, toward the N-pole of the solenoid.

When a magnet in motion is brought near a coil, a voltage is generated. This effect is *electromagnetic induction*.

When a coil is energized in proximity to a second coil an emf will be generated in the second coil as the flux is builds from 0 to its maximum level. At maximum level, the flux becomes stationary and no emf is generated in the second coil. When the coil is deenergized, the flux falls to 0 and emf is again generated in the second coil while the flux is in motion.

Nonmagnetic materials have no effect on a magnetic field.

Diamagnetic materials exhibit a very slight opposition to magnetic lines of force. They tend to be repelled from both poles of a magnet and align at right angles to the field.

Paramagnetic materials slightly assist the passage of magnetic lines of force. i.e. aluminum, platinum

Ferromagnetic materials greatly assist the passage of magnetic lines of force. These materials are used in permanent magnets and as electromagnets. i.e. iron, nickel, cobalt, ferrite

Weber (Wb) The unit of magnetic flux which, in a single-turn coil, porduces an emg of 1 V when the flux is reduced to zero at a uniform rate in 1 s.

Tesla (T) The unit of flux density in a magnetic field when 1 Wb of flux occurs in a plane of 1 m^2; i.e. 1 tesla is equal to 1 Wb/m^2. Greatest near the poles or within the magnet.

$$B = \frac{\Phi}{A}$$

where B = teslas
Φ = webers
A = m^2

Magnetomotive force (mmf) The number of turns on a coil times the number of amps equals the mmf in amps.

$$\Im = NI$$

Magnetic field strength The number of turns on a toroidal coil times the number of amps divided by the length of the magnetic path, expressed in amps/meter (A/m).

$$H = \frac{NI}{l}$$

Force on a conductor Flux density times current times length, expressed in newtons.

$$F = BIl$$

Torque on a coil of radius r in newton meters.

$$torque = BIl\,Nr$$

Hysteresis is the effect of lagging flux density relative to changes in magnetic field strength. This is due to the core material becoming magnetized and its resistance to change. This retained flux density is called remanence or residual magnetism. The magnetic field strength required to reduce the remanence to zero is called the coercive force. If a graph is drawn with *remanence* on the y-axis and *coercive force* on the x-axis, the resulting figure is called a hysteresis loop. A soft iron core material yields a narrow loop, meaning that hysteresis losses would be a minimum, making this material suitable for coils undergoing a large number of reversals per second. A hard steel core yields a wide loop, indicating a large core loss, making it unsuitable for coils undergoing reversals but is a good material for permanent magnets. A ferrite core also has a wide loop but with a more vertically compact graph approaching a square shape. This characteristic lends the material to use in *magnetic memory*.

INDUCTANCE

Lenz's Law: The induced current always develops a flux which opposes the motion or change producing the current.

Faraday's Law: The EMF induced in an electric circuit is proportional to the rate of change of flux linking the circuit.

$$e_L = \frac{\Delta\Phi}{\Delta t}$$

where e_L is in volts
$\Delta\Phi$ is in Wb
Δt is in seconds

If N is the number of turns on the secondary winding, the induced emf is

$$e_L = \frac{\Delta\Phi N}{\Delta t}$$

Self-inductance: The property in which a coil induces a counter voltage in itself as the current through it grows.

Henry (H): The SI unit of inductance. The inductance of a circuit is 1 henry when an emf of 1 V is induced by the current changing at the rate of 1 A/s.

$$L = \frac{e_L}{\Delta i / \Delta t}$$

where L is in henrys
e_L is in volts
$\Delta i / \Delta t$ is amps per second

$$L = \frac{\Delta\Phi N}{\Delta i}$$

where $\Delta\Phi$ is in Wb
N is number of turns
Δi is the change in amps

$$L = m_r m_o N^2 \frac{A}{l}$$

where m_r relative permeability of material involved (air = 1)
m_o the permeability of free space ($4\pi \times 10^{-7}$)
A is the cross-sectional area
l is the coil length

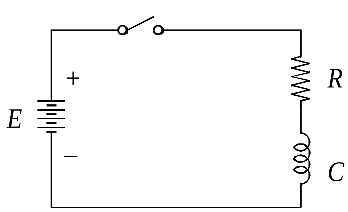
Inductors in Series:

$$L = L_1 + L_2 + L_3 + \text{L} + L_n$$

Inductors in Parallel: The total capacitance is the sum of the individual capacitances:

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \text{L} + \frac{1}{L_n}$$

Instantaneous current of a coil and resistor in series:



$$i = \frac{E}{R}\left(1 - 2.718^{-t(R/L)}\right) \quad \text{or} \quad t\frac{R}{L} = \ln\left(\frac{E}{E - iR}\right)$$

where i = instantaneous current at time t
E = supply voltage
L = inductance in henrys
t = time in seconds
R = resistance (series) in ohms

Energy stored in an inductive circuit:

$$W = \tfrac{1}{2} L I^2$$

where W is stored energy in joules
L is in henrys
I is in amperes

Mutual Inductance: When the flux from one coil cuts another adjacent (magnetically coupled) coil, an emf is induced in the second coil. The emf in the second coil sets up a flux that opposes the original flux from the first coil. The induced emf is a counter-emf and is referred to as *mutual inductance*. Two coils have a mutual inductance of 1 H when an emf of 1 V is induced in one coil by current changing at the rate of 1 A/s in the other coil. Depending on how much of the primary flux cuts the secondary, the coils may be classified as *loosely coupled* or *tightly coupled*. The amount of flux linkage is also defined in terms of a coefficient of coupling, k = *flux linkages between primary and secondary ÷ total flux produced by primary*. When both coils are wound on the same iron core, k = 1.

$$M = \frac{e_L}{\Delta i / \Delta t}$$

where M is mutual inductance in henrys
e_L is the voltage induced in the secondary coil
$\Delta i / \Delta t$ is the rate of change of current in the primary coil in amps per second

$$e_L = \frac{\Delta\Phi N_s}{\Delta t}$$

where e_L is the voltage induced in the secondary coil
$\Delta\Phi$ is the total change in flux linking with the secondary winding
N_s is the number of turns on the secondary winding
Δt is the time required for the flux change

$$M = k\sqrt{L_1 L_2}$$

where k is the coefficient of coupling
L_1 is the inductance in henrys of the primary coil
L_2 is the inductance in henrys of the secondary coil