## POLYNOMIALS

$$
f(x)=a_{n} x^{n}+a_{n-1} x^{n-1}+a_{n-2} x^{n-2}+\ldots+a_{o}
$$

## Definitions:

Powers must be non-negative whole numbers.
One independent variable only.
The leading cooefficient is the multiplier of the $x$ with the highest power.
A rational number can be put in the form of an integer divided by an integer.
A rational function is a polynomial divided by a polynomial.
The degree of the function indicates the number of possible zeros of the function.

## Some third degree polynomials factored:

$x^{3}+1=\left(x^{2}-x+1\right)(x+1)$
$x^{3}-1=\left(x^{2}+x+1\right)(x-1)$
$x^{3}+y^{3}=(x+y)\left(x^{2}-x y+y^{2}\right)$
$x^{3}-y^{3}=(x-y)\left(x^{2}+x y+y^{2}\right)$
$(x+y)^{3}=x^{3}+3 x^{2} y+3 x y^{2}+y^{3}$
Synthetic Division: The problem:

$$
x - 3 \longdiv { x ^ { 2 } + 4 x - 6 }
$$

The problem is rewritten using the root 3 from the divisor (sign changes), and the leading coefficient from each $x$-term in descending order of powers. If a power of $x$ is missing, a 0 would be substituted:

$$
\begin{array}{lllll}
3 & 1 & 1 & -18 & 18
\end{array}
$$

The coefficient of the $x$-term in the divisor, $x-3,1$ in this case, is divided into the leading coefficient of the dividend. The result is placed below the line. The number under the second coefficient is that result times the root:


The second column is added, producing 4 , which is also multiplied by the root. The result, 12, is placed in the third column:

| 1 | 1 | -18 | 18 |
| ---: | ---: | ---: | ---: |
|  | 3 | 12 |  |
| 1 | 4 |  |  |

The operation is carried out to its conclusion and the result is read from the bottom row. The rightmost value is the remainder. Second from the right is the coefficient of $x^{0}$, followed by the coefficient of $x^{1}$, etc.

| 1 | 1 | -18 | 18 |
| ---: | ---: | ---: | ---: |
|  | 3 | 12 | -18 |
| 1 | 4 | -6 | 0 |

Quadratic Equation: where $a x^{2}+b x+c=0$,

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

The root of a polynomial: Where $x-r$ is a factor of a polynomial, $r$ is a root.

Finding the root of a polynomial:

$$
P(x)=2 x^{3}+x^{2}+x-1=0
$$

If the equation has a rational root $p / q$, then $p$ is a divisor of -1 and $q$ is a divisor of 2 . Hence $p$ must be $\pm 1$ and $q$ must be $\pm 1$ or $\pm 2$. The rational root $p / q$ must be in the set $\{1,-1,1 / 2-1 /\}_{2}$ which is constructed by dividing all possible $p^{\prime s}$ all possible $q^{\prime s}$. This does not guarantee that a root exists but it does encompass all possible roots. Remember that the sign of the root is opposite of the sign as it appears in the factor. In other words, if -1 is a root, then $(a x+1)$ is a factor.
Note that this method requires that the original equation have integer coefficients. If non-integer coefficients are present, they must first be eliminated by multiplication.
So to find the factors, you divide all the possible roots into the polynomial using synthetic division. Results that yield a remainder of zero (lower right digit) indicate a root which can be converted to a factor. The other factor is read from the bottom line.

Decomposition of Fractions: A fraction with a factorable polynomial in the denominator (and in the numerator for that matter) can be broken down to a sum of simplier fractions.
The denominator is reduced to simplist terms. Terms with an internal power of 2 require an $x$-term in the numerator. Terms raised to a power require separate fractions with denominators of each power from 1 to the highest power. To determine the values of the new numerators, multiply each by the value needed to achieve the original common denominator, add them together and set this equal to the original numerator.

$$
\begin{aligned}
& \frac{\text { numerator }}{(x+a)\left(x^{2}+b\right)(x+c)^{2}}=\frac{A}{x+a}+\frac{B x+C}{x^{2}+b}+\frac{D}{x+c}+\frac{E}{(x+c)^{2}} \\
& \text { numerator }=A\left(x^{2}+b\right)(x+c)^{2}+(B x+C)(x+a)(x+c)^{2}+ \\
& \quad D(x+a)\left(x^{2}+b\right)(x+c)+E(x+a)\left(x^{2}+b\right)
\end{aligned}
$$

Descartes Rule of Signs: The number of positive real zeros of a polynomial is either equal to the number of variations in sign of the coefficients or else less than that number by a positive even integer. The number of negative real zeros of a polynomial is either equal to the number of variations in sign of $P(-x)$ or else less than that number by a positive even integer.

## Leading Coefficient Test:

Degree odd, L.C. + : Rises to right, falls to left. Degree odd, L.C. - : Rises to left, falls to right. Degree even, L.C. + : Rises to left and right. Degree even, L.C. - : Falls to left and right.

Multiplicity: $y=(x+1)^{1}(x-2)^{2}$ If the exponent is odd, the graph crosses the $x$-axis. If it is even, it only touches.

## Graphing Rational and Irrational Equations: Use these

 steps.1. Factor the numerator and denominator if possible. For an irrational equation, find the domain by locating $x$ intercepts and positive intervals of the radicand.
2. Test for symmetry.
3. List $x$ and $y$ intercepts.
4. List vertical, horizontal, and slant asymptotes.
5. Test intervals on $x$-axis to see where the function is positive or negative.

Binomial Theorem: $(x+a)^{n}=\sum_{j=0}^{n}\binom{n}{j} x^{n-j} a^{j}$, where

$$
\binom{n}{j} \text { is the binomial coefficient, }\binom{n}{j}=\frac{n!}{j!(n-j)!}
$$

Binomial Theorem Example:

$$
\begin{aligned}
& (2 y-3)^{4} \Rightarrow \\
= & {\left[\begin{array}{l}
4 \\
0
\end{array}\right)(2 y+(-3))^{4}+\binom{4}{1}(-3)(2 y)^{3}+\binom{4}{2}(-3)^{2}(2 y)^{2} } \\
& +\binom{4}{3}(-3)^{3}(2 y)+\binom{4}{4}(-3)^{4} \\
= & 1 \cdot 16 y^{4}+4(-3) 8 y^{3}+6 \cdot 9 \cdot 4 y^{2}+4(-27) 2 y+1 \cdot 81 \\
= & 16 y^{4}-96 y^{3}+216 y^{2}-216 y+81
\end{aligned}
$$

Reference: Precalculus Mathematics CTC Library QA39.2L5

