# PARTIAL FRACTIONS

**Two methods for converting a polynomial fraction into a sum**

This is useful when the fraction needs to be integrated.

## *THE METHOD OF CLEARING FRACTIONS:*

### *THE PROBLEM*

$$\frac{2v^2+1}{v^3-}$$

### *THE SOLUTION*

Factor the denominator and use each factor to form the denominators of a sum of fractions. Assign variables to the new numerators.

$$v^3 - v = v(v+1)(v-1)$$

$$\frac{A}{v} + \frac{B}{v+1} + \frac{C}{v-1} = \frac{2v^2+1}{v^3-v}$$

Take the new variables and multiply them by the denominators of the other fractions in the sum. Set the sum of these factors equal to the original numerator.

$$A(v+1)(v-1) + Bv(v-1) + Cv(v+1) = 2v^2+1$$

Carry out the multiplication:

$$Av^2 - A + Bv^2 - Bv + Cv^2 + Cv = 2v^2 + 1$$

We now collect like coefficients of the equation:

| | LEFT SIDE | RIGHT SIDE | EQUATION FORMED |
|---|---|---|---|
| COEFFICIENTS OF $v^2$ | $A+B+C$ | 2 | $A+B+C=2$ |
| COEFFICIENTS OF $v$ | $-B+C$ | 0 | $B=C$ |
| CONSTANTS | $-A$ | 1 | $A=-1$ |

Solving these equations we find: $A=-1$ $B=3/2$ $C=3/2$

### *THE RESULT*

Substituting these values we get:

$$\frac{-1}{v} + \frac{3/2}{v+1} + \frac{3/2}{v-1} = \frac{2v^2+1}{v^3-v}$$

## *THE CASE OF REPEATED FACTORS*

For example, if we have a denominator that factors to $(x + 1)(x - 2)^3$ where the factor $(x - 2)$ appears three times, we need to represent the factor in the powers of 1, 2, and 3. Our partial fractions would look like this. This applies to both of the methods examined here.

$$\frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)^3}$$

## *THE HEAVISIDE "COVER-UP" METHOD, an alternative partial fractions method:*

### *THE PROBLEM*

$$\frac{2v^2+1}{v^3-v}$$

### *THE SOLUTION*

Factor the denominator and use each factor to form the denominators of a sum of fractions. Assign variables to the new numerators.

$$\frac{2v^2+1}{v^3-v}=\frac{A}{v}+\frac{B}{v+1}+\frac{C}{v-1} \qquad \text{(eq. 1)}$$

Rewrite the problem with its denominator factored.

$$\frac{2v^2+1}{v(v+1)(v-1)}$$

Now cover-up the first factor in the denominator (indicated by an arrow here). Choose the value required to make this factor equal to zero, which in this case is just 0.

$$\frac{2v^2+1}{\underbrace{v}_{\uparrow}(v+1)(v-1)}$$

Remove the factor in question and substitute 0 for every other *v* in the expression.

$$\frac{2(0)^2+1}{(0+1)(0-1)}=\frac{1}{-1}=-1$$

The value of *A* is determined to be -1 and can be substituted into equation eq 1 above.

$$\frac{2v^2+1}{v^3-v}=\frac{\overbrace{-1}^{\downarrow}}{v}+\frac{B}{v+1}+\frac{C}{v-1}$$

Now cover-up the second factor in the denominator (indicated by an arrow here). Choose the value required to make this factor equal to zero, which in this case is -1. As before, remove the factor in question and substitute -1 for every other *v* in the expression.

$$\frac{2v^2+1}{v\underbrace{(v+1)}_{\uparrow}(v-1)} \qquad \frac{2(-1)^2+1}{-1(-1-1)}=\frac{3}{2}$$

The value of *B* is determined to be 3/2 and can be substituted into equation eq 1.

$$\frac{2v^2+1}{v^3-v}=\frac{-1}{v}+\frac{\overbrace{3/2}^{\downarrow}}{v+1}+\frac{C}{v-1}$$

Now cover-up the third factor in the denominator (indicated by an arrow here). Choose the value required to make this factor equal to zero, which in this case is 1. As before, remove the factor in question and substitute 1 for every other *v* in the expression.

$$\frac{2v^2+1}{v(v+1)\underbrace{(v-1)}_{\uparrow}} \qquad \frac{2(1)^2+1}{1(1+1)}=\frac{3}{2}$$

### *THE RESULT*

The value of *C* is determined to also be 3/2 and can be substituted into equation eq 1 .

$$\frac{2v^2+1}{v^3-v}=\frac{-1}{v}+\frac{3/2}{v+1}+\frac{\overbrace{3/2}^{\downarrow}}{v-1}$$